SPATIAL HOLE BURNING and TWO-WAVE MIXING

Coherent waves in laser produce periodic variation of gain (spatial holes burning)

This effect mitigates the efficeincy of standing-wave lasers in compare to the traveling wave amplifiers.
The generation of single longitudinal mode becomes unstable

Can we get rid of spatial hole burning?

Yb easily transfer its excitaiton to neighbors. How far?

Do we need to measure the grating of gain at scale $\lambda/2$

The non-equality of gain for strong wave and weak wave: measure locality of nonlinear response of the medium.

When are equal the gains of nonlinearly interacting waves?

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J.J.Zayhowski. Limits imposed by spatial hole burning on the single-mode operation of standing-wave cavities. – Opt.Lett., 1990, v.15, No.8, p.431-433

R.Paschotta, J.Aus der Au, G.J.Spühler, S.Erhard, A.Giesen, U.Keller. Passive mode locking of thin disk lasers: effect of spatial hole burning. – Appl.Phys.**B** 72, y.2001, p.267-278.

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COUNTER-PROPAGATING WAVES

$$(\Delta + k^2)E = i\,2k\,\beta(EE^*)\;E$$

$k = \frac{2\pi n}{\lambda}$; See [Born, Wolf, Principles of Optics, p.612, Eq.(8)]; $\beta/k \ll 1$

$$E = \mathcal{E}_1 e^{ikz} + \mathcal{E}_2 e^{-ikz}$$

$$(\mathcal{E}_1'' + 2ik\mathcal{E}_1' - \Delta_\perp \mathcal{E}_1)\,e^{ikz} + (\mathcal{E}_2'' - 2ik\mathcal{E}_2' - \Delta_\perp \mathcal{E}_2)\,e^{-ikz} =$$
$$= 2ik\beta\left(\left|\mathcal{E}_1 e^{ikz} + \mathcal{E}_2 e^{-ikz}\right|^2\right)\left(\mathcal{E}_1 e^{ikz} + \mathcal{E}_2 e^{-ikz}\right)$$

consider propagation of $\mathcal{E}_1$:

$$2ik\mathcal{E}_1' - \Delta_\perp \mathcal{E}_1 = 2ik\beta\left(\left|\mathcal{E}_1 + \mathcal{E}_2 e^{-2ikz}\right|^2\right)\left(\mathcal{E}_1 + \mathcal{E}_2 e^{-2ikz}\right)$$

Case $\beta(I) \approx \frac{g_0}{2}\left(1 - \frac{I}{I_{\mathrm{sat}}}\right)$ described by

A.E.Siegman. Lasers. – University Science Books, California, 1986, p.321:

$$\mathcal{E}_1' + \frac{\Delta_\perp}{2ik}\mathcal{E}_1 = \frac{g_0}{2}\;(1 - (|\mathcal{E}_1|^2 + 2|\mathcal{E}_2|^2)/I_{\mathrm{sat}})$$

More general case:

$$\mathcal{E}_1' + \frac{\Delta_\perp}{2ik}\mathcal{E}_1 = \int_{-\pi}^{pi} \beta\left(\left|\mathcal{E}_1 + \mathcal{E}_2 e^{-i2kz}\right|^2\right)\left(\mathcal{E}_1 + \mathcal{E}_2 e^{-i2kz}\right)\mathrm{d}(2kz)$$

$$\mathcal{E}_1' + \frac{\Delta_\perp}{2ik}\mathcal{E}_1 = G\mathcal{E}_1$$

where

$$G\mathcal{E}_1 = \tfrac{1}{2\pi}\int_{-\pi}^{\pi} \beta\left(\left|\mathcal{E}_1 + \mathcal{E}_2 e^{i\phi}\right|^2\right)\left(\mathcal{E}_1 + \mathcal{E}_2 e^{i\phi}\right) \mathrm{d}\phi$$

NONLINEARITY OF SATURATION:

$$\beta(I) = \frac{g_0/2}{1 + I/I_{\mathrm{sat}}} \; ; \qquad G\mathcal{E}_1 = \frac{\mathcal{E}_1 g_0/2}{2\pi}\int_{-\pi}^{\pi} \frac{\left(1 + \left|\frac{\mathcal{E}_2}{\mathcal{E}_1}\right| \cos\phi\right) \mathrm{d}\phi}{1 + \frac{|\mathcal{E}_1|^2}{I_{\mathrm{sat}}} + \frac{|\mathcal{E}_2|^2}{I_{\mathrm{sat}}} + \frac{2|\mathcal{E}_1\mathcal{E}_2|}{I_{\mathrm{sat}}}\cos\phi}$$

Then

$$G = \frac{g_0}{2} F\left(\frac{|\mathcal{E}_1|^2}{I_{\mathrm{sat}}}, \frac{|\mathcal{E}_2|^2}{I_{\mathrm{sat}}}\right)$$

where

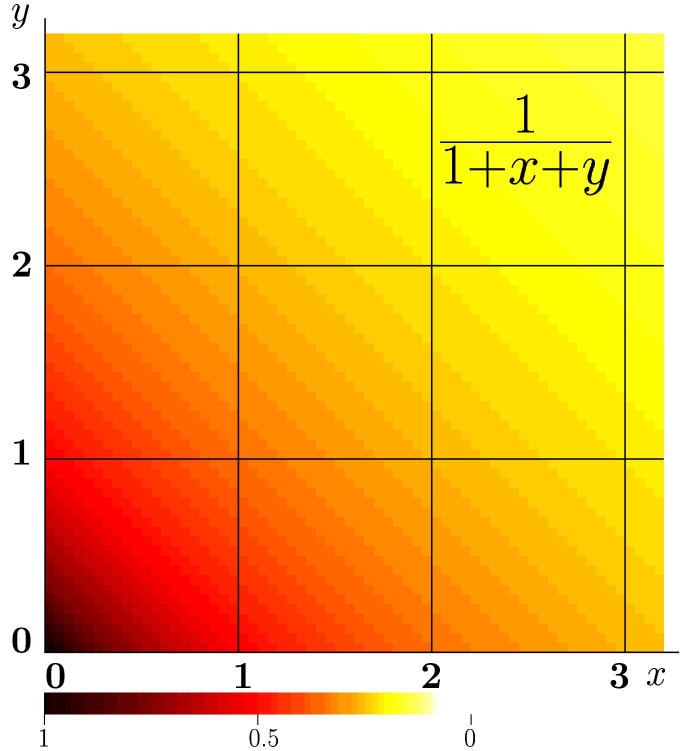
$$F(x,y) = \frac{1}{2x}\left(1 - \frac{1 - x + y}{\sqrt{1 + 2(x+y) + (x-y)^2}}\right)$$

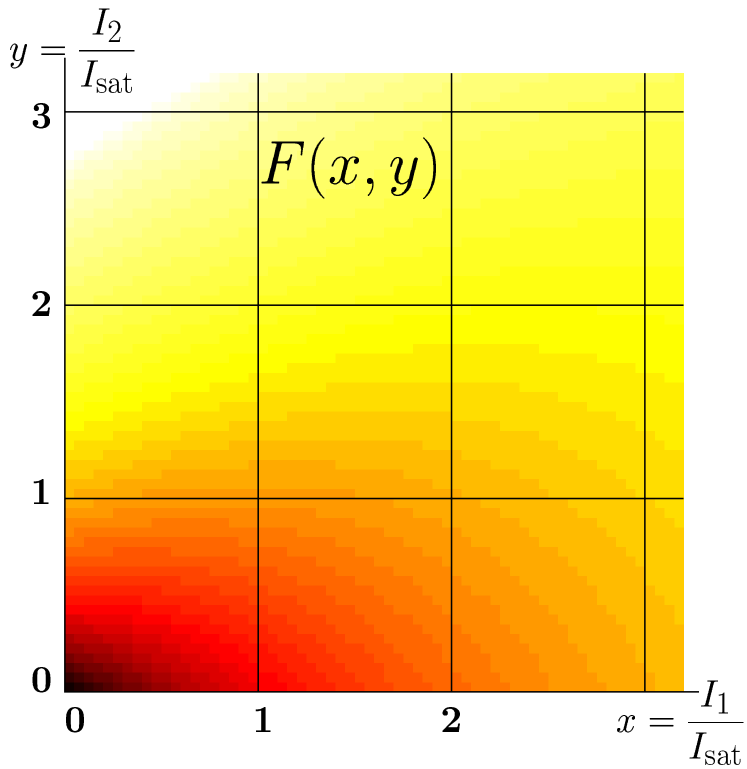
(Formula 3.613 from [Gradstein,Ryshik. Table of integrals,..]:

$$\int_0^{\pi} \frac{\cos(nx)\ \mathrm{d}x}{1 + a\cos(x)} = \frac{\pi}{\sqrt{1-a^2}}\left(\frac{\sqrt{1-a^2} - 1}{a}\right)^n$$

NONLINEARITY OF SATURATION

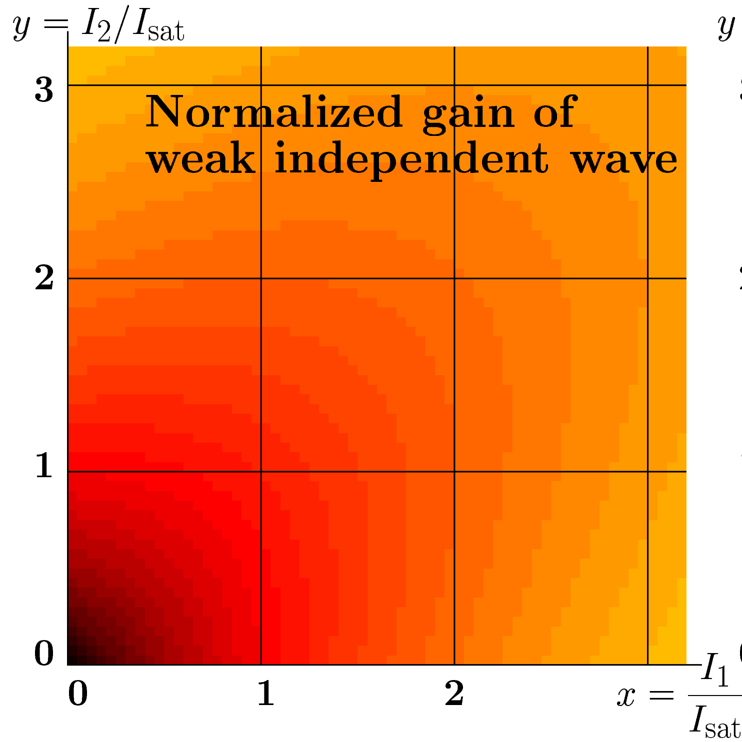
$$\mathcal{E}_1' + \frac{\Delta_\perp}{2ik}\mathcal{E}_1 = \frac{g_0}{2}\, F\left(\frac{|\mathcal{E}_1|^2}{I_{\text{sat}}}, \frac{|\mathcal{E}_2|^2}{I_{\text{sat}}}\right)\mathcal{E}_1 \quad , \qquad F(x,y) = \frac{1}{2x}\left(1 - \frac{1-x+y}{\sqrt{1+2(x+y)+(x-y)^2}}\right)$$

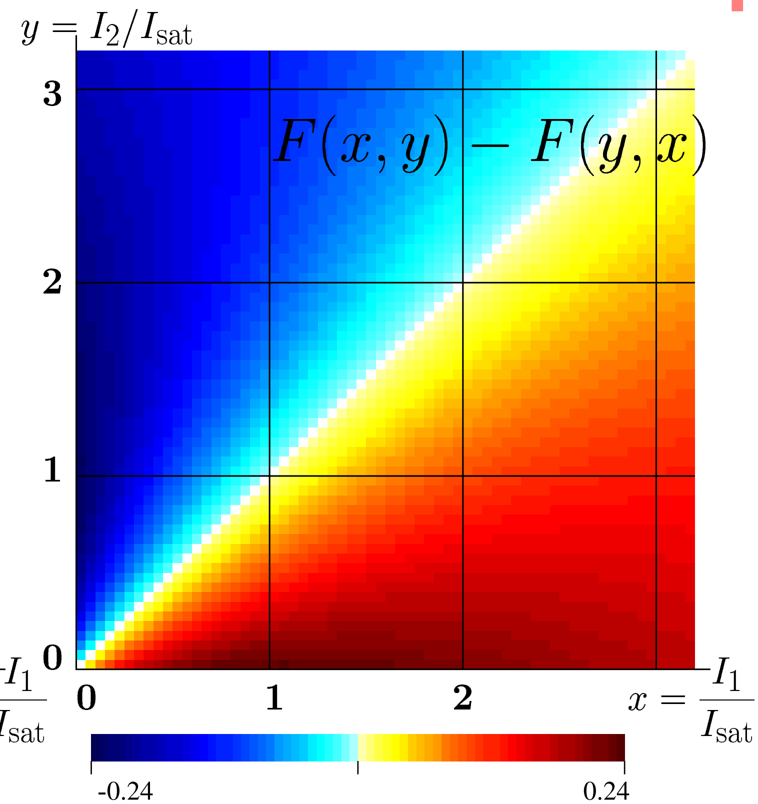




Non-local response:
Both wave gave the same gain

local response:
The strongest wave has strongest gain





EXPERIMENT ON TWO-WAVE MIXING