Usuallly one makes mirrors as flat as possible.
Flat mirrors are good for reflection of various waves:
photons, phonons.
For some applications the ridged surfaces are better.
Avoid parasite oscillation in high gain lasers.
Ridged mirrors reflect atoms better than a flat surface.
Usually one makes mirrors as flat as possible. Flat mirrors are good for reflection of various waves: photons, phonons.

For some applications the ridged surfaces are better.
Avoid parasite oscillation in high gain lasers.
Ridged mirrors reflect atoms better than a flat surface.


The quantitative prediction of the efficiency of reflection of waves from a ridged surface is the new idea which is inscribed into the world science.
The Zeno effect is kitchen where the scaling law for the reflectivity was cooked out. The speculations about Zeno effect bring no new effects which would not follow from the wave equation. You have no need to follow the sequences of hypothesis, predictions and conclusion.
You may skip all the words about Zeno effect and consider the zeno-estimate of the reflectivity as just fit of numerical simulations and experimental data.
Atomic mirrors exist
Unfortunately, flat surfaces are not so good to reflect atoms. The van der Waals interaction attracts atoms to the surface. Atoms get accelerated, and hit the surface. They feel the inhomogeneity of the mirror at the atomic and molecular scale. This causes the diffused scattering instead of the specular reflection we need for atomic optics.

The specular reflection of atoms from the flat surfaces is possible due to the short scale of the van der Waals potential. Slow atoms: \[ k = \frac{mv}{\hbar} \]

most of atomic mirrors work at incidence angle \( \approx \frac{\pi}{2} \)

Grazing angle \( \theta = \frac{\pi}{2} - \) (incidence angle)
Magneto-Optical Trap

MOT of He*

$2^3S_1$

resonant laser at 1083 nm

Movable edge

HeNe laser

screen

silicon plate

MCP detector

Micro-Channel Plate (array of detectors)

Fresnel diffraction mirror for an atomic wave.
F. Shimizu.
Specular reflection of very slow neon atoms from a solid surface.

\[ R \]

![Graph showing the relationship between \( R \) and \( v \), mm/sec with a labeled 'flat surface'.]
$L = 10 \mu m$, $\ell = 40 \text{nm}$


Fig. 1. Scanning microscope photograph of the silicon grating surface. Top: cross-sectional view. Bottom: expanded view of the ridge. The grating is fabricated on the $\{0, 0, 1\}$ surface. The ridge runs along the $\{1, 1, 0\}$ direction. The side walls of the ridge are $\{\pm 1, \mp 1, 1\}$ facets. The arrow in the top figure indicates the direction of the incident atomic beam.

F. Shimizu, J. Fujita.

Giant quantum reflection of neon atoms from a ridged silicon surface. – J. of the Physical Society of Japan, Vol. 71, No. 1 (2002), p.5-8; fig.1
F. Shimizu, J. Fujita.
Giant quantum reflection of neon atoms from a ridged silicon surface.
– J. of the Physical Society of Japan, Vol. 71, No. 1 (2002), p. 5-8; fig. 3
$L = 5 \mu m$

$\ell = 100 \text{nm}$

Fresnel diffraction mirror for an atomic wave.
**Graph (a):**

- Legend:
  - Red triangle: $\text{He}^* 81 \text{ m/s}$
  - Green square: $\text{He}^* 64 \text{ m/s}$
  - Yellow diamond: $\text{He}^* 35 \text{ m/s}$
  - Blue triangle: $\text{Ne}^* 3 \text{ m/s}$
  - Black circle: $\text{He}^*$ flat Si

- Data points for different velocities representing reflectivity at various normal incident velocities.

**Graph (b):**

- Plot of reflectivity vs. $\beta$, defined as $\beta = \sqrt{kL/\pi} \theta = \nu/\nu_0/\pi$.

- Lines and data points indicating the relationship between reflectivity and $\beta$.

---

**Equation:**

\[ \beta = \sqrt{kL/\pi} \theta = \frac{\nu}{\nu_0/\pi} \]
The Zeno effect is a class of phenomena when a transition is suppressed by some interaction which allows the interpretation of the final state in terms of “a transition has not yet occurred” or “a transition already occurred”.

We consider the transition

from the half-space $y > 0$

to the half-space $y < 0$

Reflectivity by the continuous detection

\[(\nabla^2 + k^2 + i\gamma^2\vartheta(y))\Psi = 0\]

\[k = \frac{mV}{\hbar}, \quad \gamma^2 = \frac{k}{L}\]

\[\Psi = \begin{cases} 
e^{ik_x x + ik_y y} - r e^{ik_x x - ik_y y}, & y \leq 0 \\ (1 - r) e^{ik_x x + (i\alpha - \beta)y}, & y \geq 0 \end{cases}\]

\[k_x \approx k; \quad k_y = \theta k\]

\[\alpha^2 - \beta^2 = k^2\]

\[2\alpha\beta = \gamma^2\quad \rightarrow\quad \alpha = \sqrt{\frac{1}{2} (\sqrt{k^4 + \gamma^4 + k^2})}
\]

\[\beta = \sqrt{\frac{1}{2} (\sqrt{k^4 + \gamma^4 - k^2})}\]

\[r = \frac{i\alpha - \beta + ik}{i\alpha - \beta - ik}\]

\[rr^* = R_z(p) = \frac{\sqrt{1/p^4 + 1 + 1 - \sqrt{2}}}{\sqrt{1/p^4 + 1 + 1 + \sqrt{2}}}\]

\[p = \sqrt{kL} \theta = \sqrt{\frac{mL}{\hbar V}} \theta\]

\[R_z(p) = \exp(-\sqrt{8p}) \pm 2\%\]
Dimensionless momentum: \[ p = \sqrt{kL \theta} = \sqrt{\frac{mL}{hV}} \theta \]

Zeno estimate: \[ R_z(p) = \frac{\sqrt{\frac{1}{p^4} + 1 + 1 - \sqrt{2}}}{\sqrt{\frac{1}{p^4} + 1 + 1 + \sqrt{2}}} \]

Exponential fit: \[ R_z(p) = \exp\left(-\sqrt{8p}\right) \pm 2\% \]
Paraxial propagation

\[ \nabla^2 \psi + k^2 \Psi = 0 \]

\[ k_x = k \cos(\theta) \approx k - k\theta^2/2 \]
\[ k_y = -k \sin(\theta) \approx -k\theta \]

The incident field \( \Psi_{in}(x, y) = \exp(ik_xx + ik_yy) \)

\[ \Psi = \psi(x, y)e^{ikx} \]

\[ 2ik \frac{\partial}{\partial x} \psi + \frac{\partial^2}{\partial y^2} \psi = 0 \]

Let the distance \( L \) between ridges be unit of length.
\( X = x/L \), \( Y = y\sqrt{k/L} \); \( \psi(x, y) = E(X, Y) \)
\[ E_{\text{in}}(X, Y) = e^{-ipY-i(p^2/2)X} \]

\[ p = \sqrt{kL \theta} \]

\[ 2i \frac{\partial}{\partial X} E + \frac{\partial^2}{\partial Y^2} E = 0 \]

\[ E(X, Y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-iqY-i(q^2/2)X} F(q) \, dq \]

\[ E \left( X, \left(n - \frac{N}{2}\right) d \right) \rightarrow E_n(X) \]

\[ F \left( \left(n - \frac{N}{2}\right) d \right) \rightarrow F_n \]

\[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \ldots \, dq \quad \rightarrow \quad \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \ldots \]


Analysis of scattered field:

\[ f(q) = \frac{1}{\sqrt{2\pi}} \int e^{-iqY} E(X, Y) \exp\left(-\frac{Y^2}{2W^2}\right) dY \]

\[ q_m = \sqrt{p^2 + 4\pi m} \]

\[ R_m = \frac{|f(q_m)|^2}{W^2} \]

Trend:

\[ R_m \approx \text{fit}(q_m) = \frac{1}{(1 + q_m)^4} \]


\[ \lg(R_m(p)) \]

\[ q_m(p) = \sqrt{4\pi m + p^2} \]
\[
S_m(p) = R_m(p)^{q_m(p)/p}
\]

\[
S_0(p) = R_0(p) \approx \exp(-1.68(1+0.018p^2)p \pm 2\%), \quad p < 3.6
\]

\[
\tilde{s}_8(p) = \sum_{m<8} S_m(p) \approx \exp(-1.547(1+0.01p^2)p \pm 2\%) \, , \quad p < 3.6
\]
Trend: $R_m(p) \approx \frac{1}{(1+\sqrt{p^2+4\pi m})^4}$ ; $q_m(p) = \sqrt{p^2 + 4\pi m}$
Use of symmetry

\[ F(1, Y) = F(0, Y) e^{i(Y-Z)^2} \]

\[ F(1, Y) = F_{\text{ini}}(X, Y) = \sum_m r_m e^{i q_m Y - (i/2) q_m^2 X} \]

\[ q_m = \sqrt{p_m^2 + 4\pi m} \]

Using the Green function (Kirhgof integral):

\[ F(1, Y) = \frac{1}{\sqrt{2i\pi}} \int_0^\infty F(0, Y) e^{i(Y-Z)^2} dZ \]

\[ e^{-ipY} - \sum_m r_m e^{iq_m Y} = \frac{1}{\sqrt{2i\pi}} \int_0^\infty \left( e^{-ipX} e^{i(Y-Z)^2} - \sum_m r_m e^{i(Y-Z)^2} \right) dZ \]

\[ \sum_m \left( e^{iq_m Y} - \frac{1}{\sqrt{2i\pi}} \int_0^\infty e^{i(Y-Z)^2} dZ \right) r_m = e^{ipY} - \frac{1}{\sqrt{2i\pi}} \int_0^\infty e^{ipZ} e^{i(Y-Z)^2} dZ \]

\[ \sum_m e^{-k_m Y} \text{erfc} \left( \frac{Y - ik}{1+i} \right) r_m - e^{-ipY} \text{erfc} \left( \frac{Y + p}{1+i} \right) = 0 \]

where \( k_m = \sqrt{-4\pi m - p^2} \)


Residual \( \Phi = \int_0^\infty \sum_m e^{-kmY} \text{erfc} \left( \frac{Y-ik}{1+i} \right) r_m - e^{-ipY} \text{erfc} \left( \frac{Y+p}{1+i} \right) \right) dY \)

can be written as \( \Phi = r^*_m A_{m,n} r_n - r_m B^*_m - r^*_m B_m + C \)

where
\[
A_{m,n} = A(k^*_m, k_n)
\]
\[
B_m = A(k^*_m, ip)
\]
\[
B^*_m = A(-ip, k_m)
\]
\[
C = A(-ip, ip)
\]

\[ A(u, v) = \int_0^\infty e^{-(u+v)x} \text{erfc} \left( \frac{x+iu}{1-i} \right) \text{erfc} \left( \frac{x-iv}{1+i} \right) dx \]

Minimumization of the residual: \( \Phi = r^\dagger A r - r^\dagger B - B^\dagger r + C \)

replace \( r \to r + \delta r \)

\( \delta \Phi = (\delta r)^\dagger A r + r^\dagger A \delta r + (\delta r)^\dagger B - B^\dagger \delta r \)

\( \delta \Phi = (\delta r)^\dagger (A r - B) + (A r - B)^\dagger (\delta r) \)

\[ \rightarrow \quad Ar = B \]
\[ r = A^{-1} B \]

Each choice of number of terms with positive and negative \( m \) gives the estimates of reflectivities \( |r_m|^2 \).
Zero-order estimate: \( m = n = 0 \)

\[
A_{0,0} = \int_{0}^{\infty} \text{erfc}\left(\frac{x - p}{1 - i}\right) \text{erfc}\left(\frac{x - p}{1 + i}\right) \, dx
\]

\[
B_0 = \int_{0}^{\infty} e^{-2ip} \text{erfc}\left(\frac{x + p}{1 - i}\right) \text{erfc}\left(\frac{x - p}{1 + i}\right) \, dx
\]

\[
C = \int_{0}^{\infty} \text{erfc}\left(\frac{x + p}{1 - i}\right) \text{erfc}\left(\frac{x + p}{1 + i}\right) \, dx
\]

We get the 0-th order estimate

\[
r_0(p) = \frac{B_0}{A_{0,0}}
\]

Also, if \( \Phi = r^\dagger \, A \, r - r^\dagger \, B - B^\dagger \, r + C \approx 0 \)

and \( Ar = B \),

then \( r \approx C/B^* \)

(independent estimate)
Zero order estimate

dashed: lg of the Zeno-estimate $R_z(p) = \frac{\sqrt{\sqrt{\frac{1}{p^4+1}-2}}}{\sqrt{\sqrt{\frac{1}{p^4+1}+2}}}$

Circles: numerical simulations.

black solid: lg of exponential fit $\exp(-1.68(1+0.018p^2)p)$
\[ R_m = |r_m|^2; \quad r_m = (A^{-1})_{m,n} B_n; \quad -2 \leq m, n \leq 3 \]
\[ R_m = |r_m|^2; \quad r_m = (A^{-1})_{m,n} B_n; \quad -2 \leq m, n \leq 3 \]
Phase of the specular reflection

Atomic holograms:
Can we adjust both the amplitude and the phase of the reflected wave?
Variation $\delta h$ of height of the ridges $\rightarrow \delta \phi = 2\delta h \sin(\theta)$
The amplitude can be adjusted with distance $L$ between ridges. $L$ affects also the phase through $p = \sqrt{kL\theta}$

![Graph showing the relationship between $\text{Re}(r_0)$ and $\text{Im}(r_0)$ for different values of $p$.]

- $p = 1$
- $p = 2$
- $p = 3$
The curves and experimental dots seem to agree.
The curves and experimental dots seem to agree. However, the fit of simulations is a bit above. What is wrong with simulations?
\[ p = \sqrt{kL} \theta \]

\[ R_0(p) = e^{-1.68(1+0.018p^2)p} \]

\[ R_z(p) = \frac{\sqrt{\sqrt{\frac{1}{p^4}+1}+1-\sqrt{2}}}{\sqrt{\frac{1}{p^4}+1}+1+\sqrt{2}} \]
$p = \sqrt{kL \theta}$

$\text{Ne}^*\quad V = 3\text{m/sec}$

$L = 100\mu m$
$\ell = 1\mu m$

$L = 100\mu m$
$\ell = 11\mu m$

\[ R_0(p) = e^{-1.68(1+0.018p^2)p} \]

\[ R_2(p) = \frac{\sqrt{1/p^4+1+1-\sqrt{2}}}{\sqrt{1/p^4+1+1+\sqrt{2}}} \]
The Zeno-estimate can be fitted with exponential

\[ R_z(p) = \frac{\sqrt{\sqrt{1/p^4 + 1 + 1} - \sqrt{2}}}{\sqrt{1/p^4 + 1 + 1} + \sqrt{2}} \approx \exp(-2.8p) \]

At \( p < 1 \), the fit of numerical analysis gives \( R_0 \approx \exp(-1.7p) \)

Some effect gives a factor \( \sim \exp(-1.1p) \)

Van der Waals potential \( U = U(x, y) \approx C_4/y^4 \)

Can it be responsible for such a factor?

<table>
<thead>
<tr>
<th>( m )</th>
<th>( C_4 )</th>
<th>( v )</th>
<th>( V )</th>
<th>( \ell )</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>amu</td>
<td>( 10^{-56} \frac{J}{m^4} )</td>
<td>m/sec</td>
<td>m/sec</td>
<td>nm</td>
<td>( \mu m )</td>
</tr>
<tr>
<td>Ne*</td>
<td>20</td>
<td>12</td>
<td>0.019 : 0.374</td>
<td>3</td>
<td>40 : 11000</td>
</tr>
<tr>
<td>He*</td>
<td>4</td>
<td>8</td>
<td>0.034 : 1.535</td>
<td>34 : 126</td>
<td>100</td>
</tr>
</tbody>
</table>
The contribution to the Kirhhoff integral comes from the vicinity of ridges. 

Size of this region: \( s_0 = \frac{1}{k \theta} \)

This region generates the reflected wave of intensity \( R_0(\sqrt{kL \theta}) \)

Let \( y(x) \) be trajectory. The defasing due to the refraction index becomes strong at

\[
\int U(x, y(x)) \frac{dx}{V} = \hbar
\]

\[
V = \frac{k\hbar}{m}
\]

How close to the ridge should the atom pass in order to feel the ridge?

\[
\int \frac{C_4}{s^4} \frac{dx}{V} = \hbar
\]

This gives \( \frac{C_4}{s^4} D = \hbar V \) where \( D \) is length the atom passes in vicinity of the ridge. For the estimate, we assume \( D \approx \ell + 2s \). Then

\[
s = \left( \frac{(\ell + 2s)C_4}{\hbar V} \right)^{1/4}
\]
The reflected wave is attenuated proportionally the width $s$ of the ”defased” region,

$$R(p, \ell) = \left(1 - \frac{s}{s_0}\right)R_0(p)$$

What about the case $s > s_0$? Better, \( R(p, \ell) = \exp\left(-\frac{s}{s_0}\right)R_0(p) \)

$$s_0 = \frac{1}{\theta k} = \frac{\hbar}{mV\theta}$$

$$R(p, \ell) = \exp\left(-\sqrt{\frac{mV}{\hbar L}} \left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} \right) R_0(p)$$

$$s = \left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} \approx \left(\frac{(\ell+0)C_4}{\hbar V}\right)^{1/4}, \quad p = \sqrt{kL\theta}, \quad k = mV/\hbar$$

The correction coefficient \( \sqrt{\frac{mV}{\hbar L}} \left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} \) \( \sim (\ell V)^{1/4} \) has values of order of unity; it is just that we were looking for.

**Warning:** The speculations above is not a rigorous deguction. It is rather a kitchen which leads to the good fit of experimental data.

The only excuse is the comparizon with experiments.
\begin{align*}
\text{Ne}^* \quad V &= 3 \text{m/sec} \\
L &= 100 \mu\text{m} \\
\ell &= 1 \mu\text{m} \\
\ell &= 11 \mu\text{m}
\end{align*}

\begin{align*}
R(p, \ell) &= \exp \left( -\sqrt{\frac{mV}{\hbar L}} \left( \frac{(\ell + 2s)c_4}{\hbar V} \right)^{1/4} p \right) R_0(p) \\
R_0(p) &= e^{-1.68(1+0.018p^2)p} \\
R_z(p) &= \frac{\sqrt{\sqrt{1/p^4+1+1}-\sqrt{2}}}{\sqrt{\sqrt{1/p^4+1+1}+\sqrt{2}}}
\end{align*}
The only two sets of data show significant deviation from the estimate
\[ R(p, \ell) = \exp \left( -\sqrt{\frac{mV}{\hbar L}} \left( \frac{\ell+2sC_4}{\hbar V} \right)^{1/4} p \right) R_0(p) \]
$\text{He}^*$

$L = 5 \mu m$

$\ell = 100 \text{ nm}$

$V = \begin{cases} 
\times & 34 \text{ m/sec} \\
\triangle & 63 \text{ m/sec} \\
\diamond & 81 \text{ m/sec} \\
\cdot & 126 \text{ m/sec} 
\end{cases}$

$C_4 = 8 \times 10^{-56} \text{ J/m}^4$

$R_0(p) = e^{-1.68(1+0.018p^2)p}$

$R_z = \frac{\sqrt{\sqrt{1/p^4+1}+1+\sqrt{2}}}{\sqrt{1/p^4+1}+1+\sqrt{2}}$

$R(p, \ell) = \exp\left(-\sqrt{\frac{mV}{hL}}\left(\frac{(\ell+2s)C_4}{hV}\right)^{1/4} p\right) R_0(p)$
Reflection of atoms from ridged surfaces was treated with optical methods.

Ridged surface as a detector: the continuous detection would give

\[ R_z = \sqrt{\frac{\sqrt{1/p^4+1+1} - \sqrt{2}}{\sqrt{1/p^4+1+1} + \sqrt{2}}} \], \quad p = \sqrt{kL \theta} 

The more detailed description of narrow risges leads to the empiric fit

\[ R_0(p) \approx \exp(-1.68(1+0.018p^2)p \pm 2\%) \], \quad p < 3.6

Correction due to the van der Waals interaction:

\[ R(p, \ell) = \exp \left( -\sqrt{\frac{mV}{\hbar L}} \left( \frac{\ell + 2s}{\hbar V} C_4 \right)^{1/4} \right) R_0(p) \]

The estimates \( R_z(p) \) and \( R(p, \ell) \) show good agreement with experiments.

The most of formulas of this paper are not specific for atomic waves; the estimates should work as follows for waves of any other origin (photons, neutrons, phonons; even oceanic waves at the surface of water). The analysis suggested can be used in the design of ridged mirrors for various applications including atomic optics.

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