THEORY OF RIDGED MIRRORS

Dmitrii Kouznetsov, Hilmar Oberst, Fugio Shimizu, Kazuko Shimizu, Jun-ichi Fujita

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Usuallly one makes mirrors as flat as possible.
Flat mirrors are good for reflection of various waves:
photons, phonons.

For some applications the ridged surfaces are better.
Avoid parasite oscillation in high gain lasers.
Ridged mirrors reflect atoms better than a flat surface.

H.Oberst, D.Kouznetsov, K.Shimizu, J.Fujita, F.Shimizu.
Fresnel diffraction mirror for an atomic wave. Phys.Rev.Lett., in press

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F.Shimizu, J.Fujita. Giant quantum reflection of neon atoms from a ridged silicon surface. – J.Phys.Soc.Jpn.**71**(2003)p.5-8.

H.Oberst, D.Kouznetsov, K.Shimizu, J.Fujita, F.Shimizu. Fresnel diffraction mirror for an atomic wave. – Phys.Rev.Lett. (2005), in press

H.-R.Noh, K.Shimizu, F.Shimizu Imaging of an atomic beam with electrostatic lenses. – Phys. Rev. **A61** (2000) 041601 R

J.Fujita, F.Shimizu. Atom manipulation using atomic de Broglie waves. – Materials Science and Engineering **B 96**, No.2, (2002), p.159-163

The quantitative prediction of the efficiency of reflection of waves from a ridged surface is the new idea which is inscribed into the world science.
The Zeno effect is kitchen where the scaling law for the reflectivity was cooked out. The speculations about Zeno effect bring no new effects whigh would not follow from the wave equation.
You have no need to follow the sequences of hypothesis, predictions and conclusiond.
You may skip all the words about Zeno effect and consider the zeno-estimate of the reflectivity as just fit of numerical simulations and experimental data.

Atomic mirrors exist

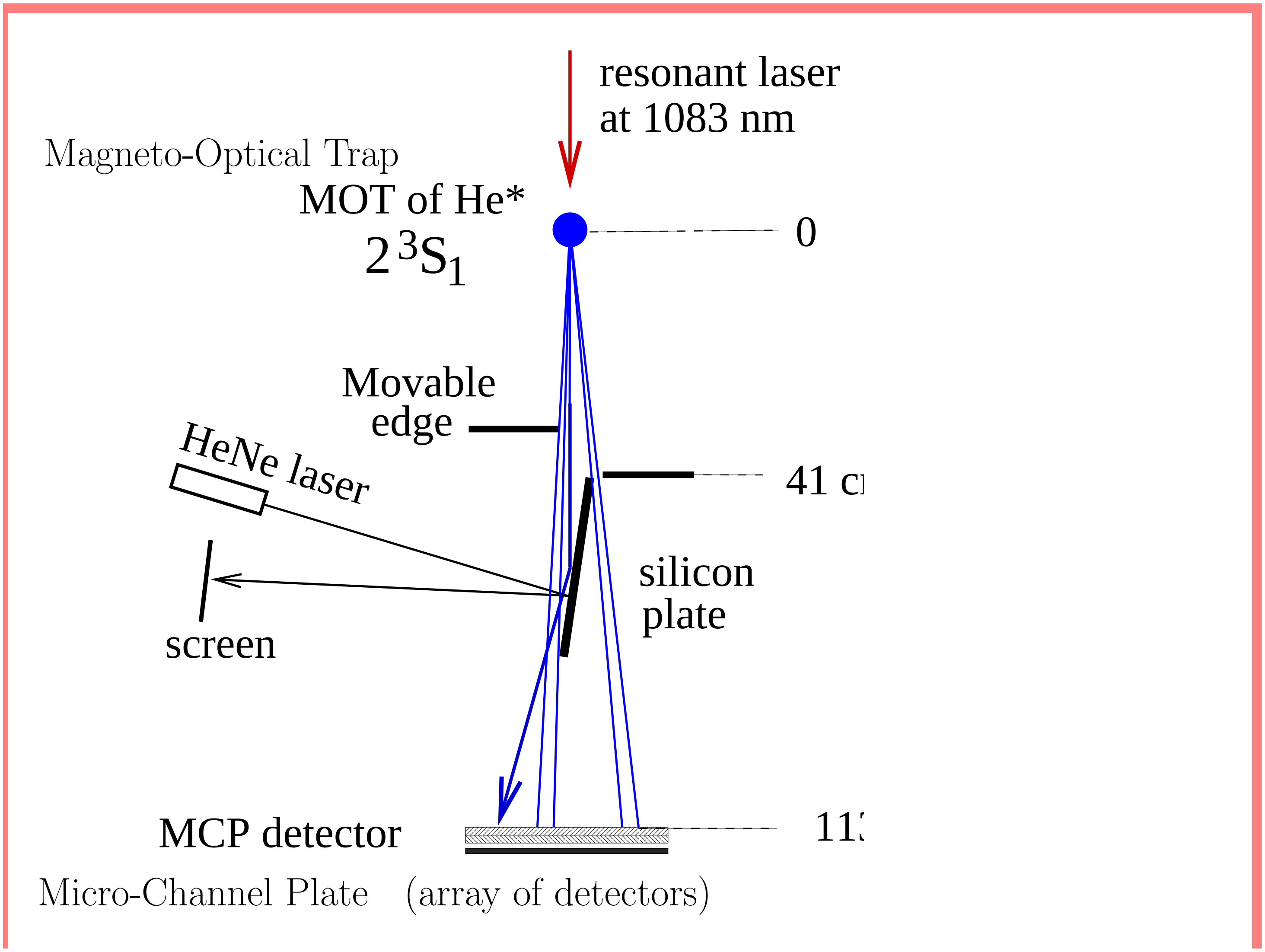
Unfortunately, flat surfaces are not so good to reflect atoms.
The van der Waals interaction attracts atoms to the surface.
Atoms get accelerated, and hit the surface.
They feel the inhomogeneity of the mirror at the atomic and molecular scale. This causes the diffused scattering instead of the specular reflection we need for atomic optics.

The specular reflection of atoms from the flat surfaces is possible due to the short scale of the van der Waals potential.

Slow atoms: $k = \frac{mv}{\hbar}$

most of atomic mirrors work at incidence angle $\approx \frac{\pi}{2}$

Grazing angle $\theta = \frac{\pi}{2} -$ (incidence angle)

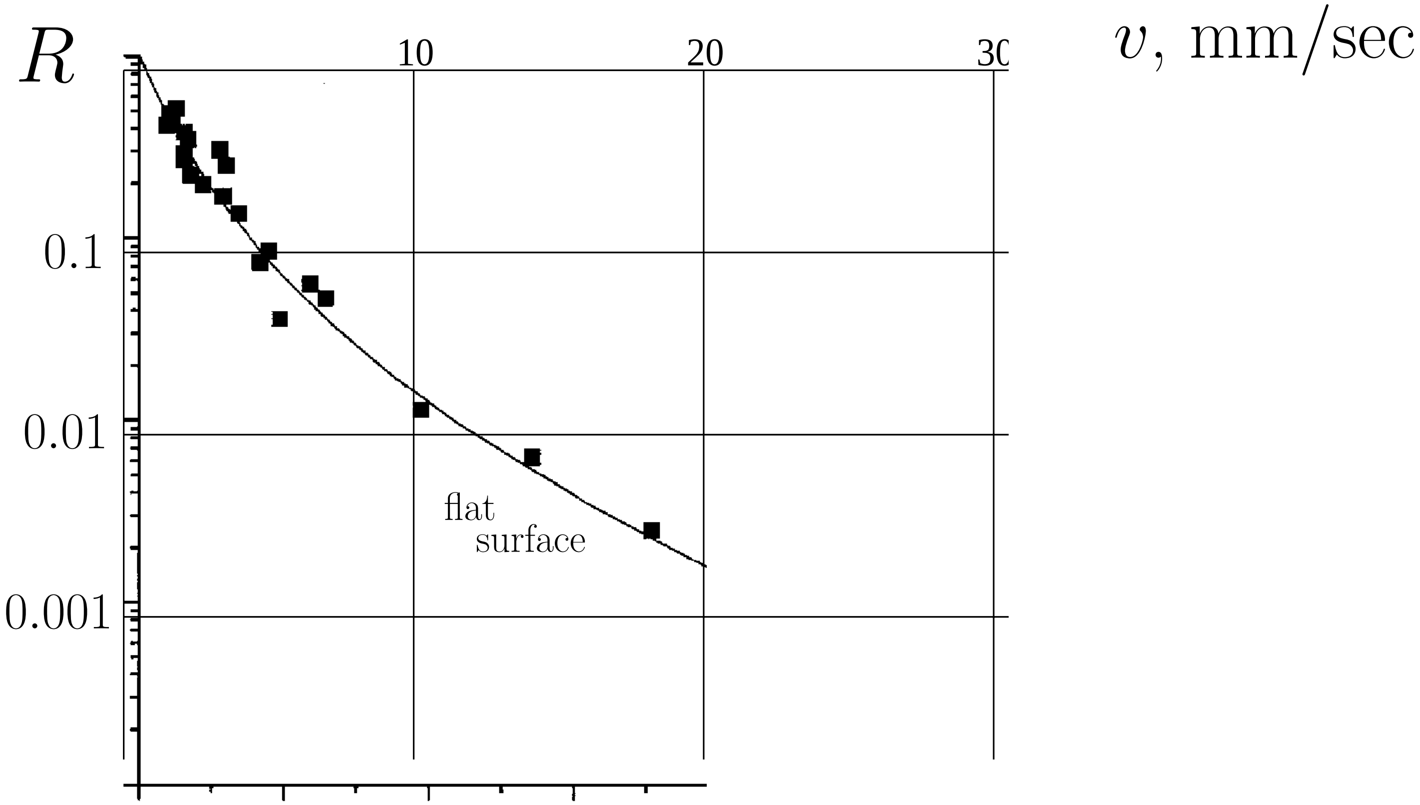


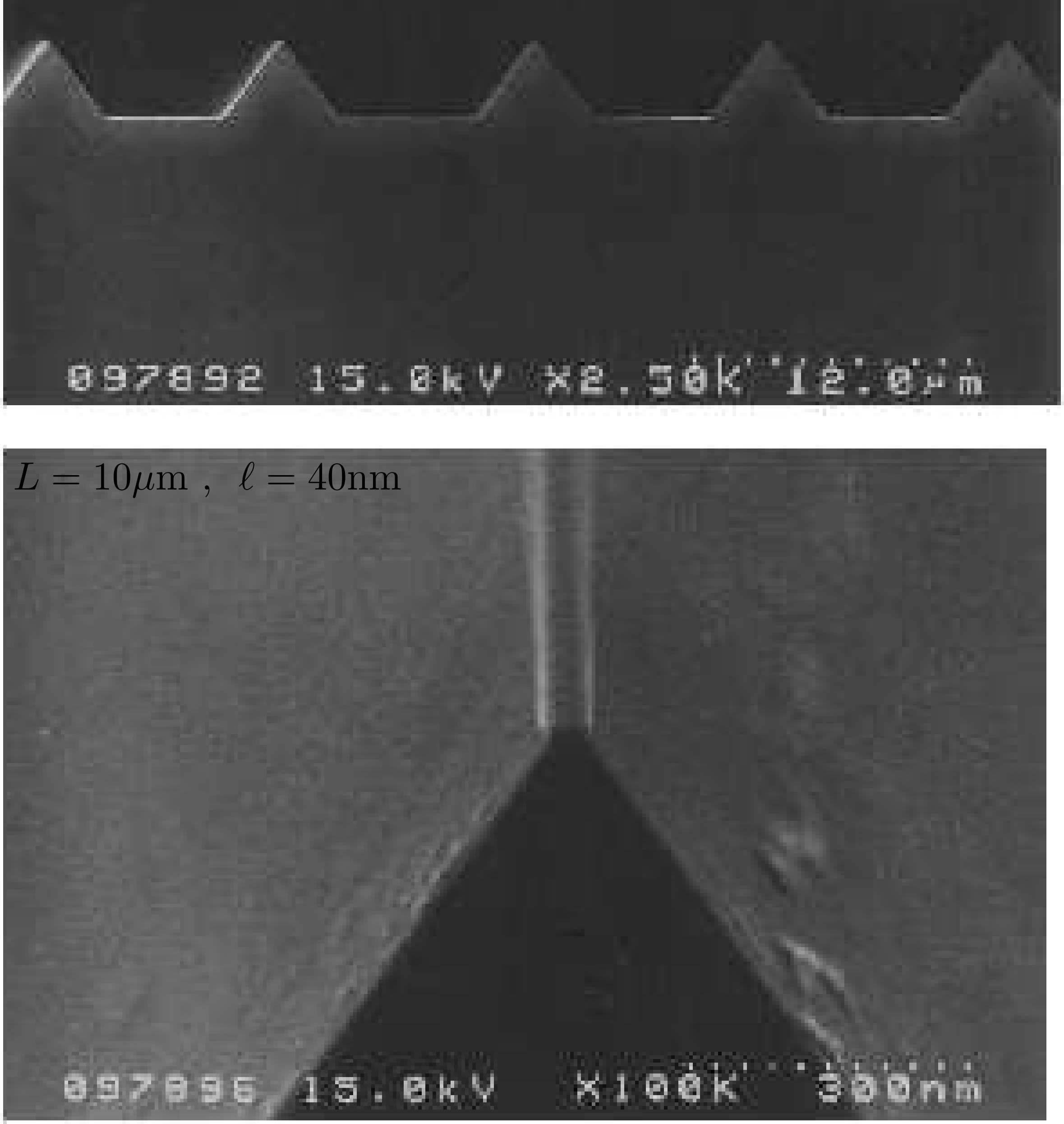
H.Oberst, D.Kouznetsov, K.Shimizu, J.Fujita, F.Shimizu.
Fresnel diffraction mirror for an atomic wave.
– Phys. Rev. Lett., 2005, in press

F.shimizu.

Specular reflection of very slow neon atoms from a solid surface.

– Phys.Rev.A, v.86, No.6, p.987-990





F.shimizu, J.Fujita, J.of the Phys.Soc. of J. 71, (2002), p.5-8; fig.1

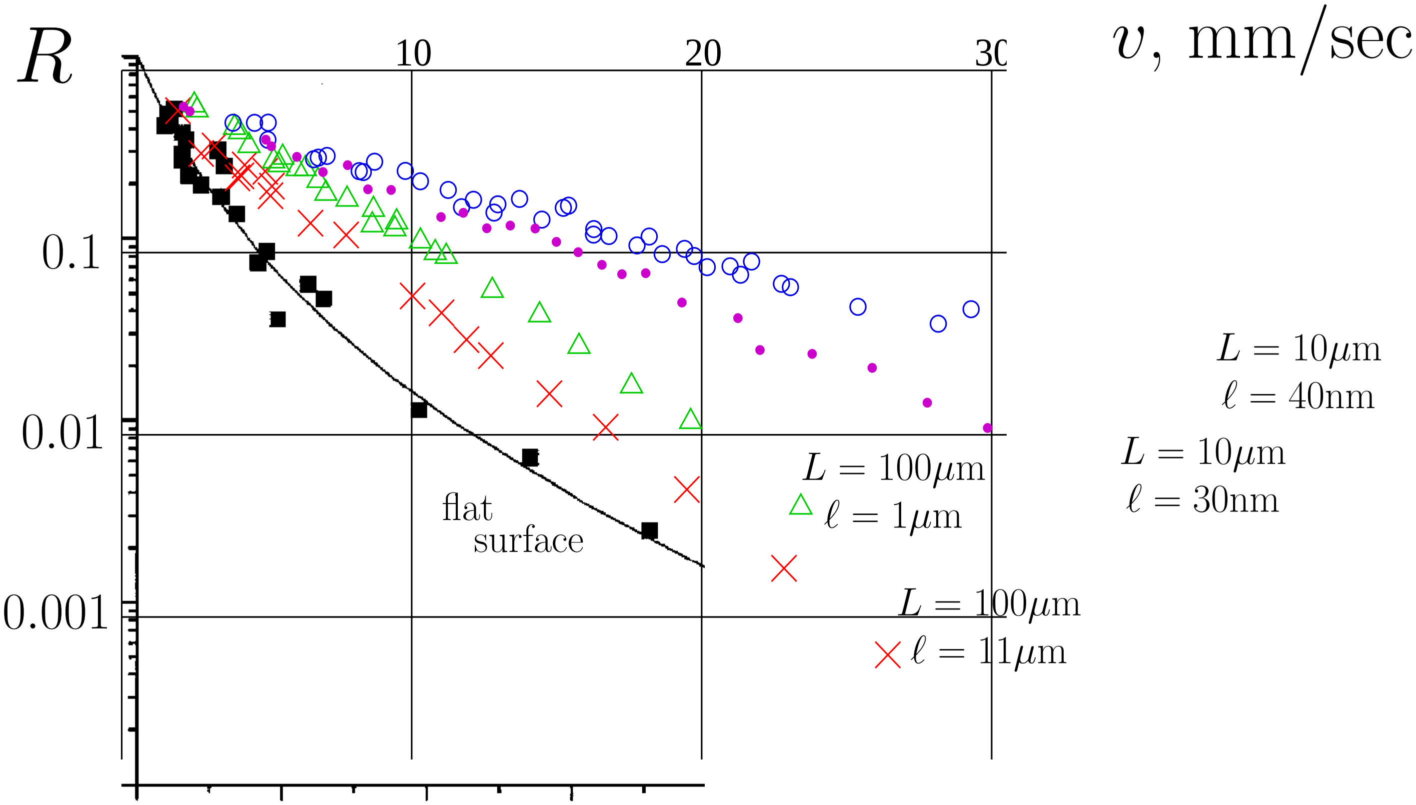
Fig. 1. Scanning microscope photograph of the silicon grating surface. Top: cross-sectional view. Bottom: expanded view of the ridge. The grating is fabricated on the (0, 0, 1) surface. The ridge runs along the (1, 1, 0) direction. The side walls of the ridge are (±1, ∓1, 1) facets. The arrow in the top figure indicates the direction of the incident atomic beam.

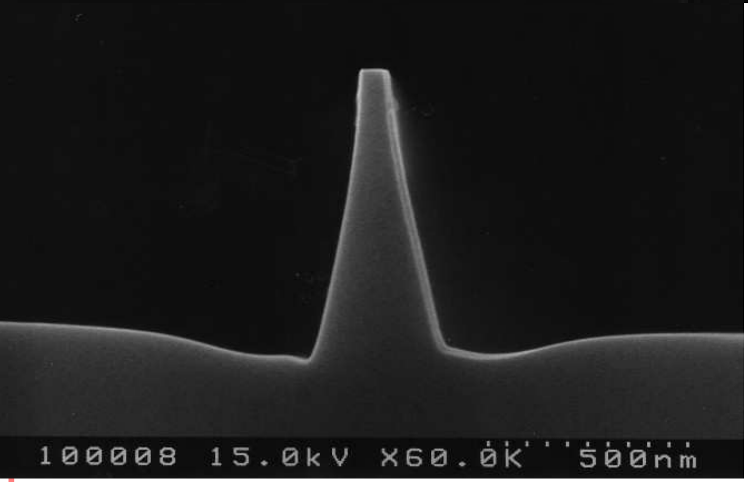
F.shimizu, J.Fujita.
Giant quantum reflection of neon atoms from a ridged silicon surface.
– J.of the Physical Society of Japan, Vol.71, No.1 (2002), p.5-8; fig.1

F.shimizu, J.Fujita.

Giant quantum reflection of neon atoms from a ridged silicon surface.

– J.of the Physical Society of Japan, Vol.71, No.1 (2002), p.5-8; fig.3





$$L = 5\mu\text{m}$$

$$\ell = 100\text{nm}$$

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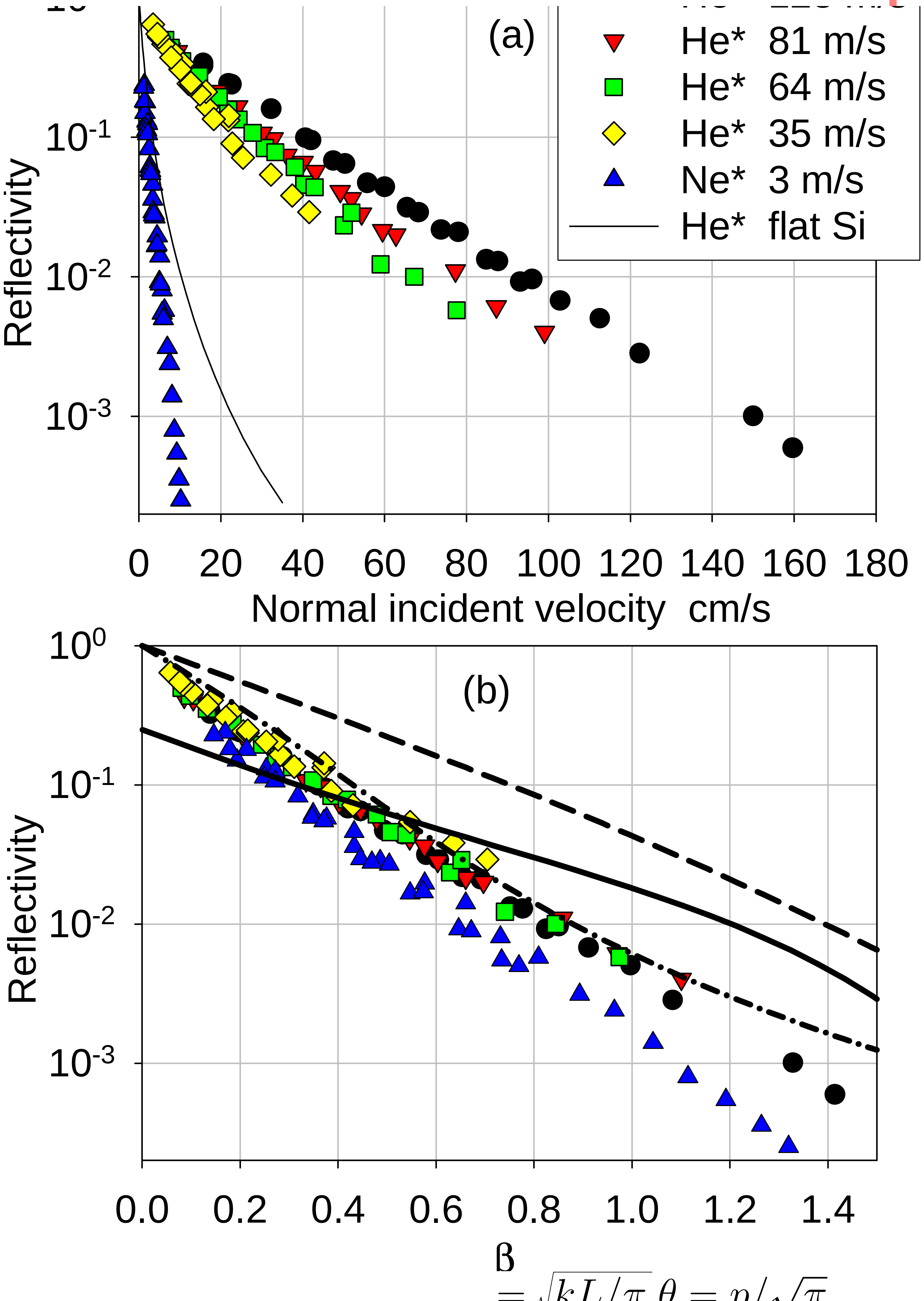
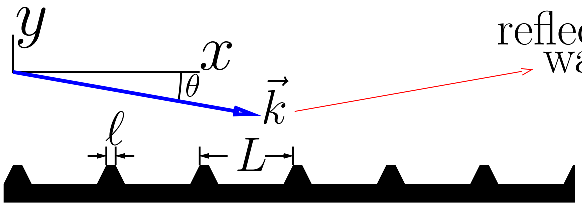
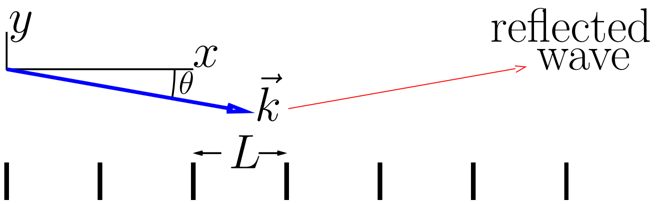

(a)
He* 81 m/s
He* 64 m/s
He* 35 m/s
Ne* 3 m/s
He* flat Si
Reflectivity
10^{-1}
10^{-2}
10^{-3}
0
20
40
60
80
100
120
140
160
180
Normal incident velocity cm/s
(b)
10^{0}
10^{-1}
10^{-2}
10^{-3}
Reflectivity
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
β

Zeno effect. Simple estimate

Ridges as detectors. Absorbing half-space with absorption $\frac{1}{L}$.





The Zeno effect is class of phenomena when a transition is suppressed by some interaction which allows the interpretation of the final state in terms of "a transition has not yet ocurred" or "a transition already ocurred".

We consider the transition

from the half-space $y > 0$
to the half-space $y < 0$

B.Misra and E.C.G.Sudarshan. The Zeno's paradox in quantum theory. – J.Math.Phys. **18** (1977) 756.

W.M.Itano, D.J.Heinzen, J.J.Bollinger, D.J.Wineland. Quantum Zeno effect. – Phys.Rev.**A 41** (1990) 2295.

B.Mielnik: The screen problem. – Foundations of Physics **24** (1994) 1113.

M.C.Fischer, B.Gutiérrez-Medina and M.G.Raizen, Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System. – Phys.Rev.Lett. **87** (2001) 040402.

S.K.Sekatskii. Fluorescence resonance energy transfer and quantum Zeno effect. – Phys. Lett. A **317**, issue 1-2 (2003) p.1-5

S.Luo. Spatial quantum Zeno effect. – Physica A, **317** (2003) 509-516

K.Koshino, A.Shimizu. Quantum Zeno and anti-Zeno effects by indirect measurement with finite errors. – Phys.Rev.**A 87** (2003), 042101.

Reflectivity by the continuous detection

$$(\nabla^2 + k^2 + i\gamma^2\vartheta(y))\Psi = 0$$

$$k = \frac{mV}{\hbar}\ , \quad \gamma^2 = \frac{k}{L}$$

$$\Psi = \begin{cases} e^{ik_{\rm x}x + ik_{\rm y}y} - r e^{ik_{\rm x}x - ik_{\rm y}y} & ,\ y \le 0 \\ (1-r)e^{ik_{\rm x}x + (i\alpha-\beta)y} & ,\ y \ge 0 \end{cases}$$

$$k_{\rm x} \approx k\ ;\quad k_{\rm y} = \theta k$$

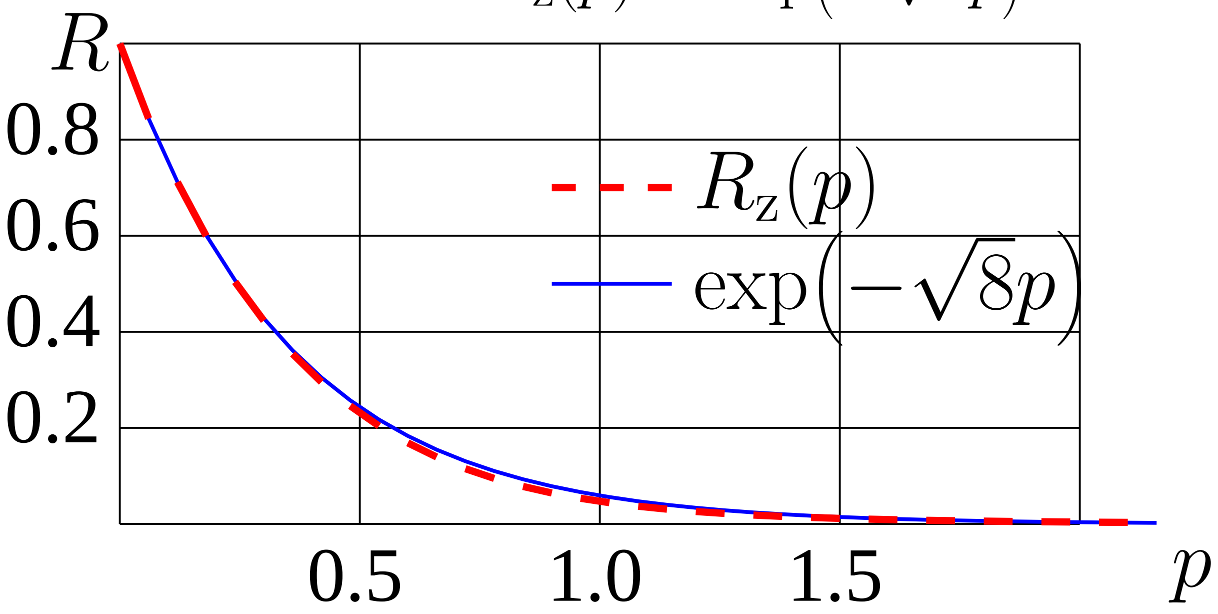
$$\begin{array}{rcl} \alpha^2 - \beta^2 & = & k^2 \\ 2\alpha\beta & = & \gamma^2 \end{array} \quad \longrightarrow \quad \begin{array}{l} \alpha = \sqrt{\frac{1}{2}\left(\sqrt{k^4+\gamma^4} + k^2\right)} \\ \beta = \sqrt{\frac{1}{2}\left(\sqrt{k^4+\gamma^4} - k^2\right)} \end{array}$$

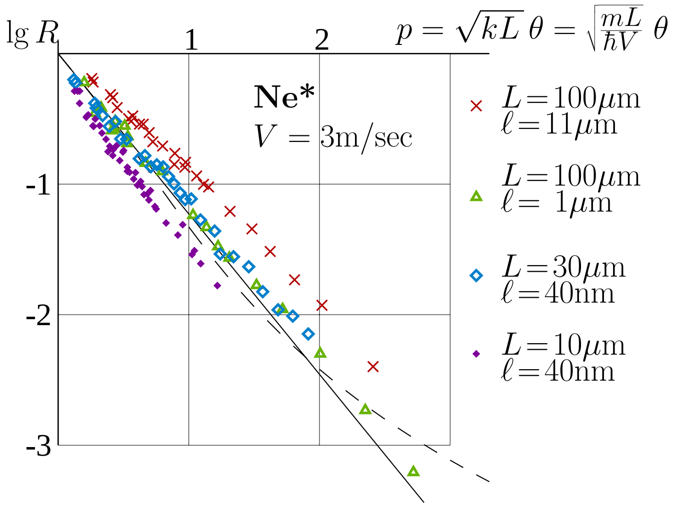
$$r = \frac{i\alpha - \beta + ik}{i\alpha - \beta - ik}$$

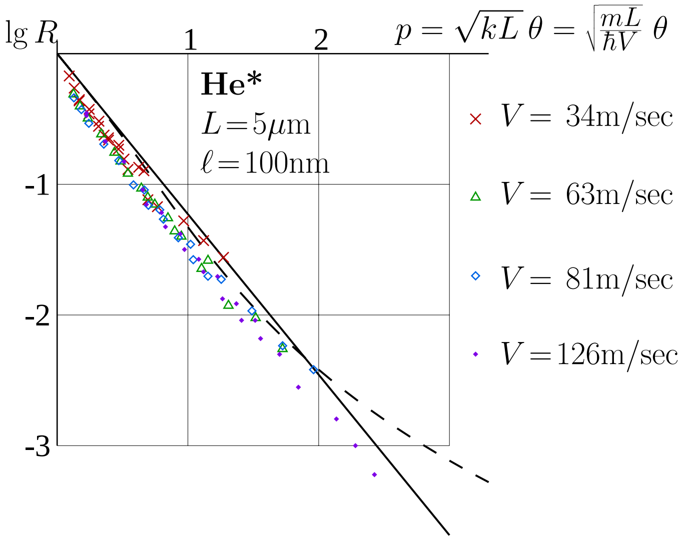
$$rr^* = R_{\rm z}(p) = \frac{\sqrt{\sqrt{1/p^4+1}+1} - \sqrt{2}}{\sqrt{\sqrt{1/p^4+1}+1} + \sqrt{2}}$$

$$p = \sqrt{kL}\ \theta = \sqrt{\frac{mL}{\hbar V}}\ \theta$$

$$R_{\rm z}(p) = \exp(-\sqrt{8}p) \pm 2\%$$







Dimensionless momentum: $p=\sqrt{kL}\,\theta=\sqrt{\frac{mL}{\hbar V}}\,\theta$

Zeno estimate: $R_{\text{z}}(p)=\frac{\sqrt{\sqrt{1/p^4+1}+1-\sqrt{2}}}{\sqrt{\sqrt{1/p^4+1}+1+\sqrt{2}}}$

Exponential fit: $R_{\text{z}}(p)=\exp\left(-\sqrt{8}p\right)\pm 2\%$

Paraxial propagation

$$\nabla^2\psi + k^2\Psi = 0$$

$k_{\mathrm{x}} = k\cos(\theta) \approx k - k\theta^2/2$.$k_{\mathrm{y}} = -k\sin(\theta) \approx -k\theta$,
The incident field $\Psi_{\mathrm{in}}(x,y) = \exp(ik_{\mathrm{x}}x + ik_{\mathrm{y}}y)$

$$\Psi = \psi(x,y)e^{ikx}$$

$$2ik\frac{\partial}{\partial x}\psi + \frac{\partial^2}{\partial y^2}\psi = 0$$

Let the distance L between ridges be unit of length.
$X = x/L$, $Y = y\sqrt{k/L}$; $\psi(x,y) = E(X,Y)$

$$E_{\mathrm{in}}(X,Y)=e^{-ipY-i(p^2/2)X}$$

$p=\sqrt{kL}\,\theta$

$$2i\frac{\partial}{\partial X}E+\frac{\partial^2}{\partial Y^2}E=0$$

$$E(X,Y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{-iqY-i(q^2/2)X}\,F(q)\,\mathrm{d}q$$

$$E\left(X,\ \left(n-\frac{N}{2}\right)d\right)\ \rightarrow\ E_n(X)$$

$$F\left(\ \left(n-\frac{N}{2}\right)d\right)\ \rightarrow\ F_n$$

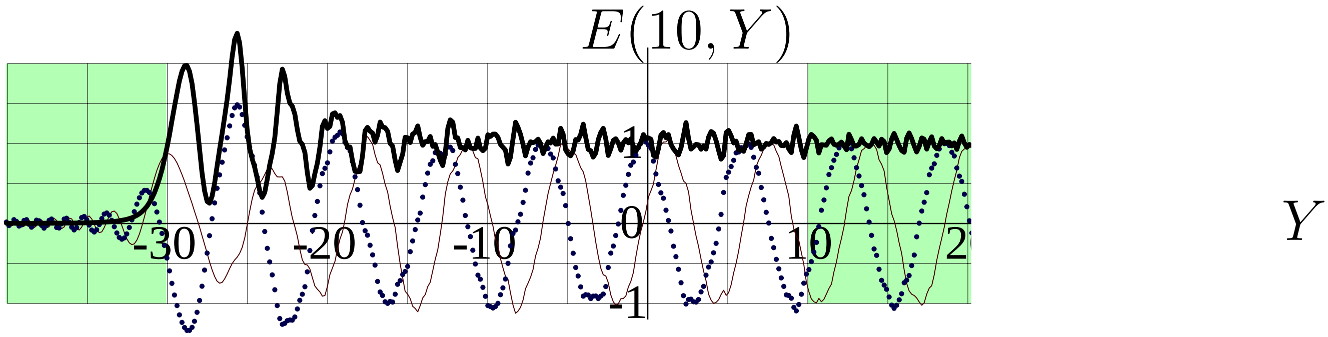
$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\ldots\,\mathrm{d}q\ \longrightarrow\ \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\ldots$$

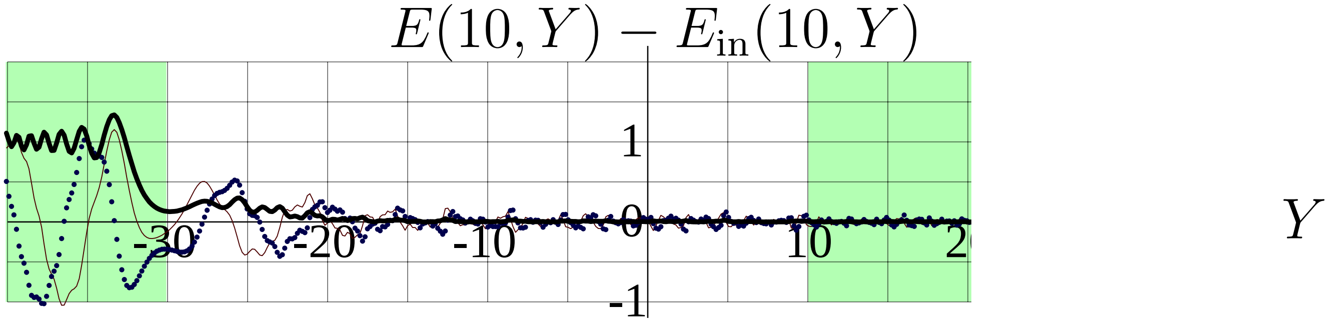
D.Yu.Kuznetsov. Transformation of the spatial structure of the monochromatic radiation in the non-linear media.
– Optics and Lasers. ed: G.G.Petrash, Nova Science Publishers,Inc., 1995; (Trydy FIAN, 1991, v.211, p.102-160)

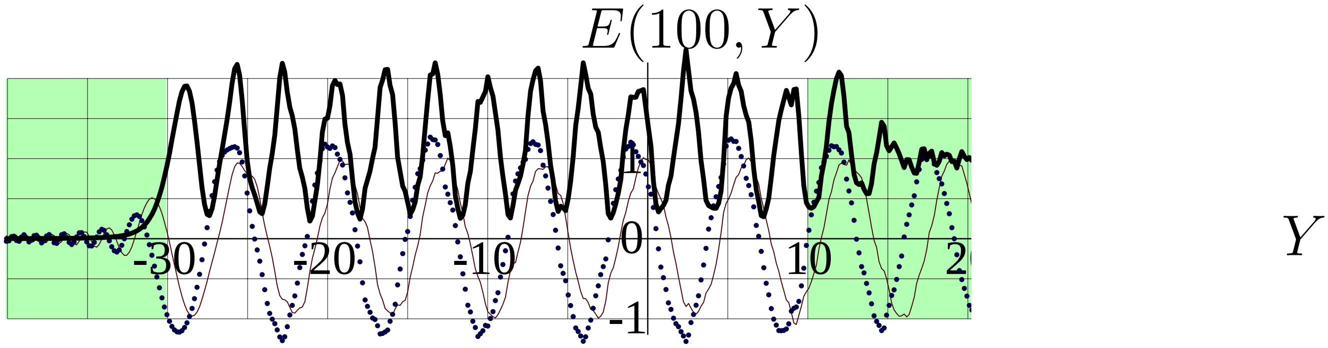
V.V. Voitsekhovich, D.Kouznetsov, D.Kh.Morozov. Density of turbulence-induced phase dislocations. – Appl. Opt. **37**, p.4525-4535 (1998)

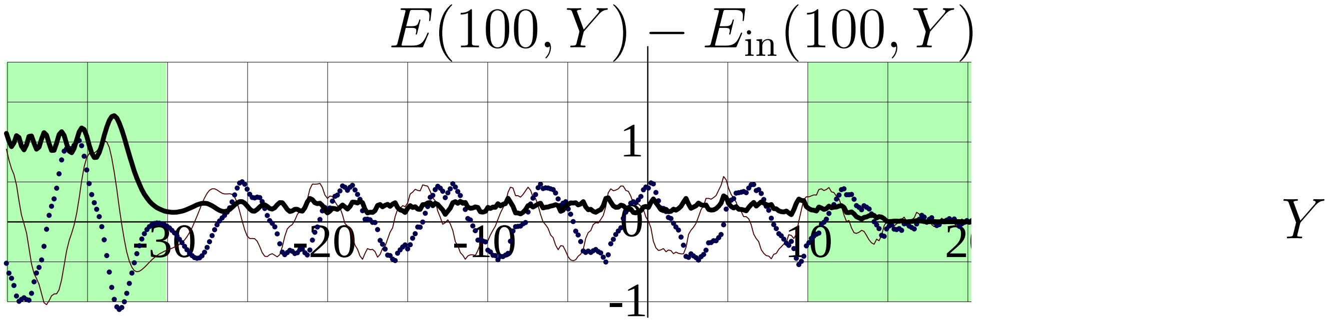
D.Kouznetsov, J.V.Moloney. Efficiency of pump in the double-clad fiber amplifiers. 2. Broken circular symmetry. – J.Opt.Soc.Am. **B 19**, No.6, p.1259-1263 (2002).

J.Moloney, A.Newell. Nonlinear Optics. Western Press, 2004.


$E(10, Y)$

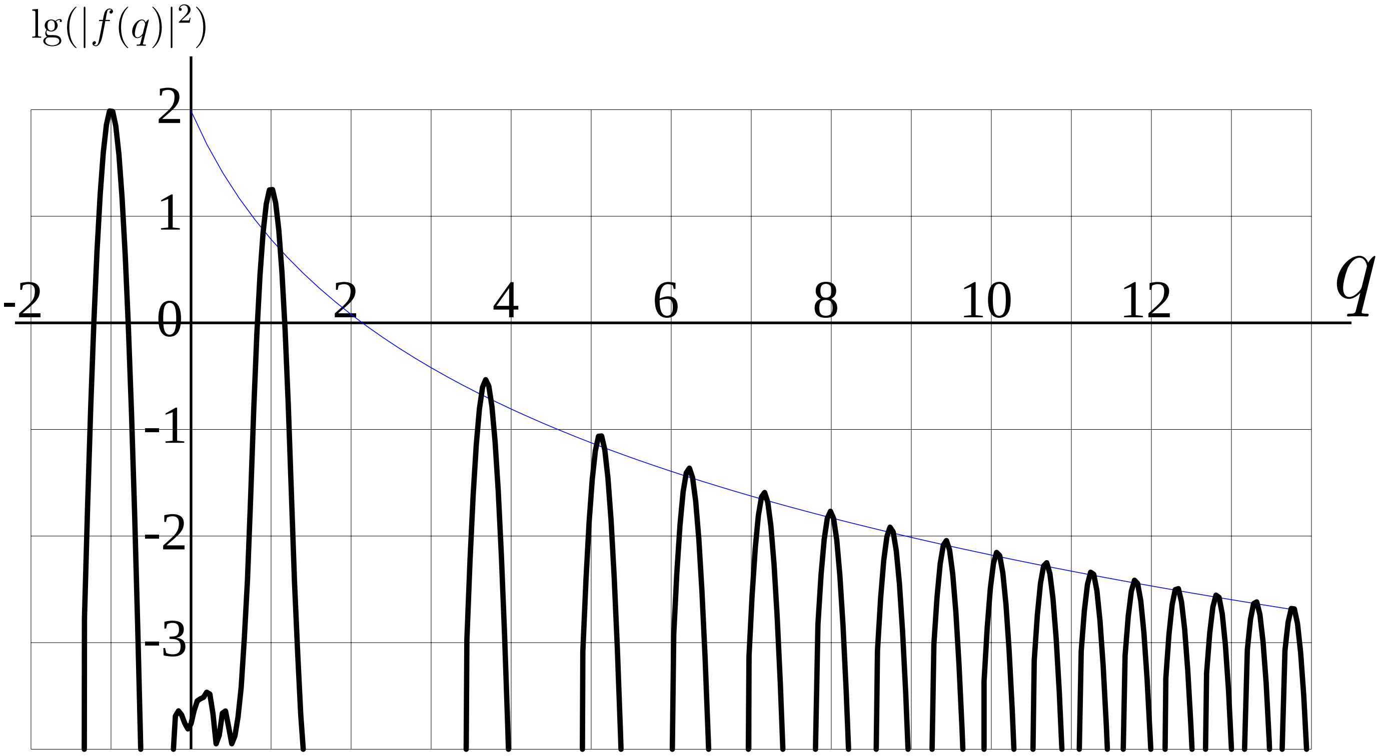

$E(10, Y) - E_{\text{in}}(10, Y)$


$E(100, Y)$


$E(100, Y) - E_{\text{in}}(100, Y)$

Analysis of scattered field:

$$f(q) = \frac{1}{\sqrt{2\pi}} \int e^{-iqY} E(X,Y) \exp\left(-\frac{Y^2}{2W^2}\right) \,\mathrm{d}Y$$



$W = 10\,,\; X = 800\,,\; N = 4096\,;$

$$q_m = \sqrt{p^2 + 4\pi m}$$

$$R_m = |f(q_m)|^2 / W^2$$

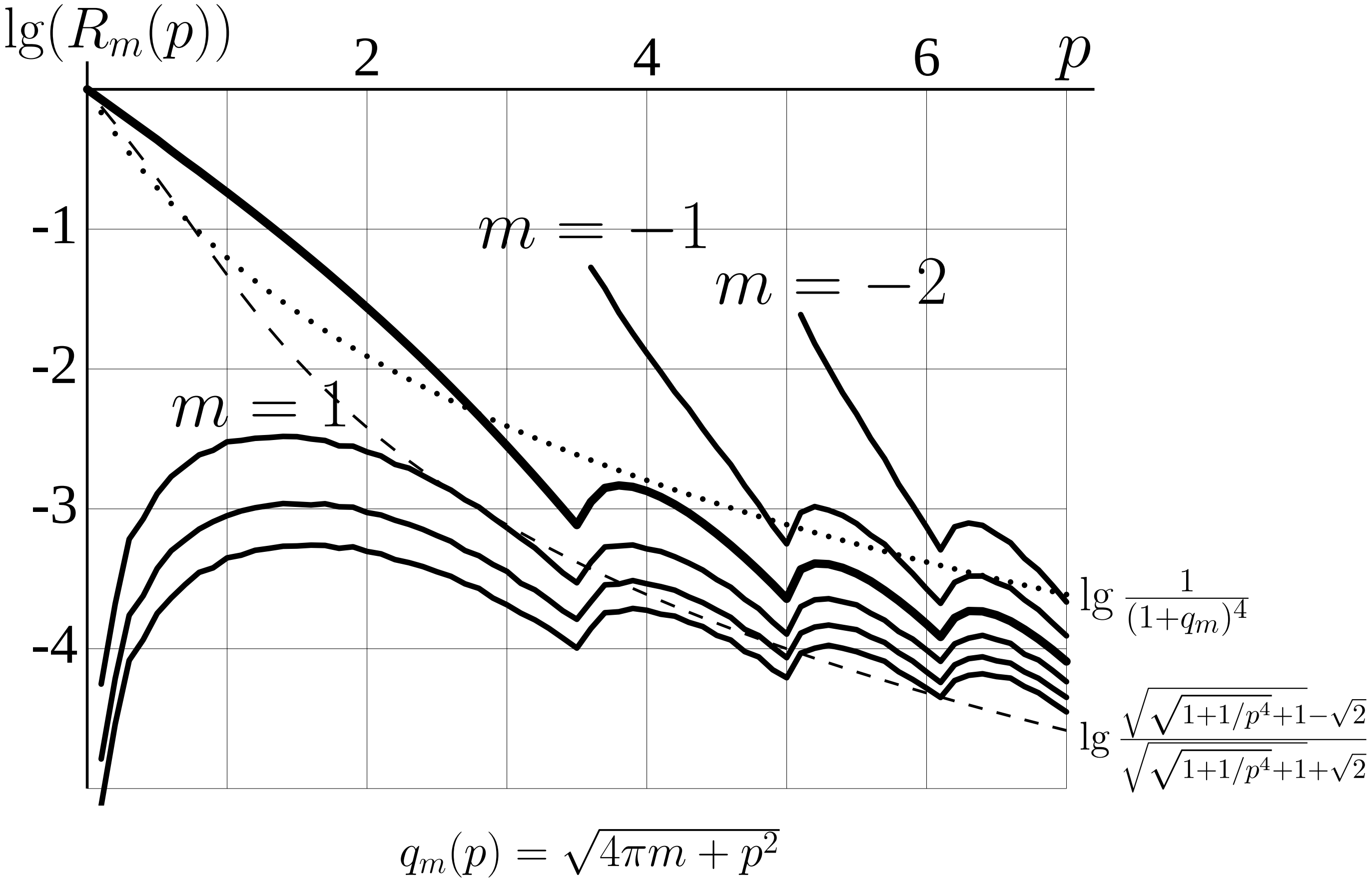
Trend:

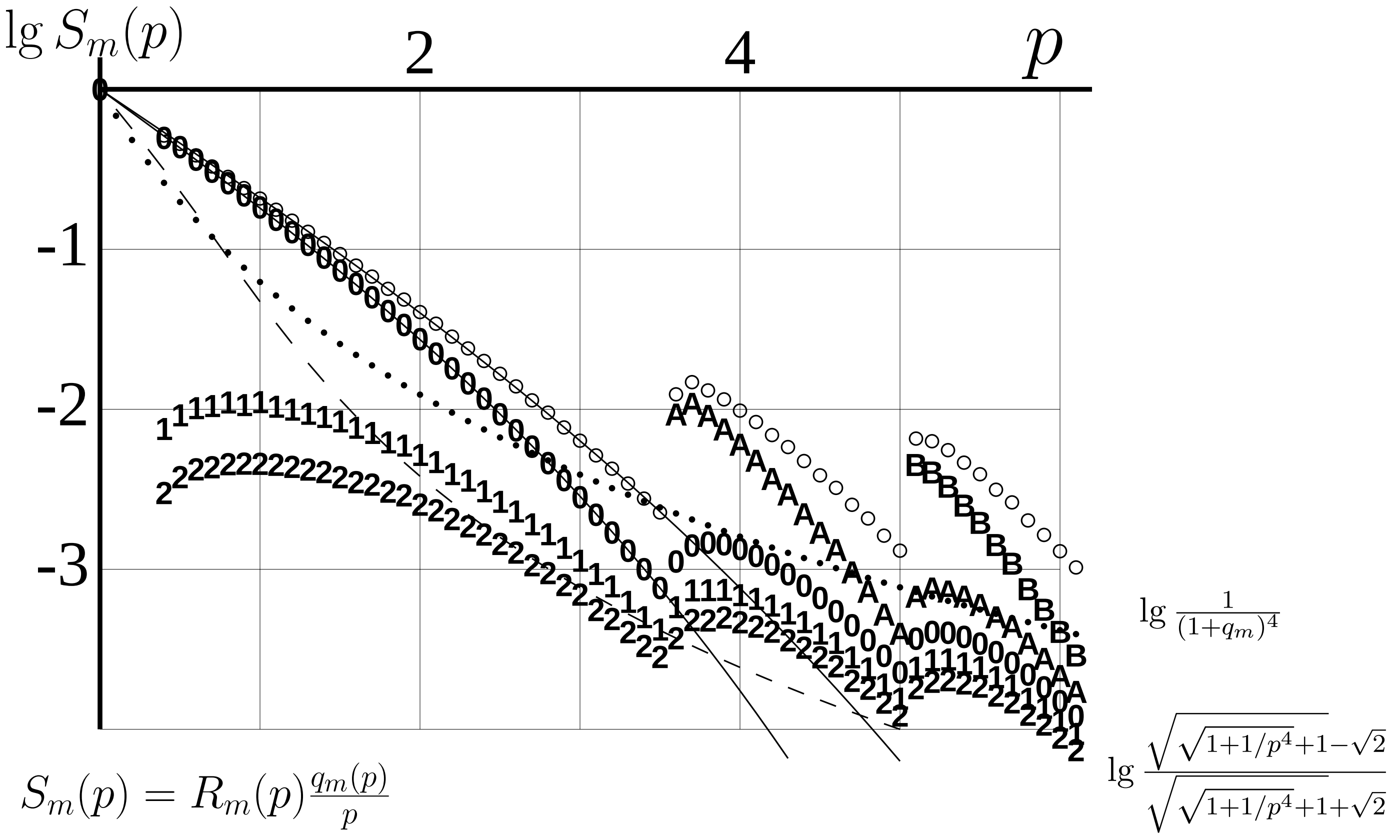
$$R_m \approx \mathrm{fit}(q_m) = \frac{1}{(1+q_m)^4}$$

P. Sheng, R. S. Stepleman, P. N. Sanda. Exact eigenfunctions for square-wave grating: application to diffraction and surface-plasmon calculations. – Phys. Rev. **B 26** (1982) p.2907-2916

P. C. Logoftu, S. A. Coulombe, B. K. Minhas, J. R. McNeil. Identity of the cross-reflection coefficients for symmetric surface-relief grating – J.Opt.Soc.Am. **A 15**, (1999),p. 1108-1114.

H.Jia, K.Yasumoto. A novel formulation of the Fourier model method in S-matric form for arbitrary chaped gratings. – International Journal of Infrared

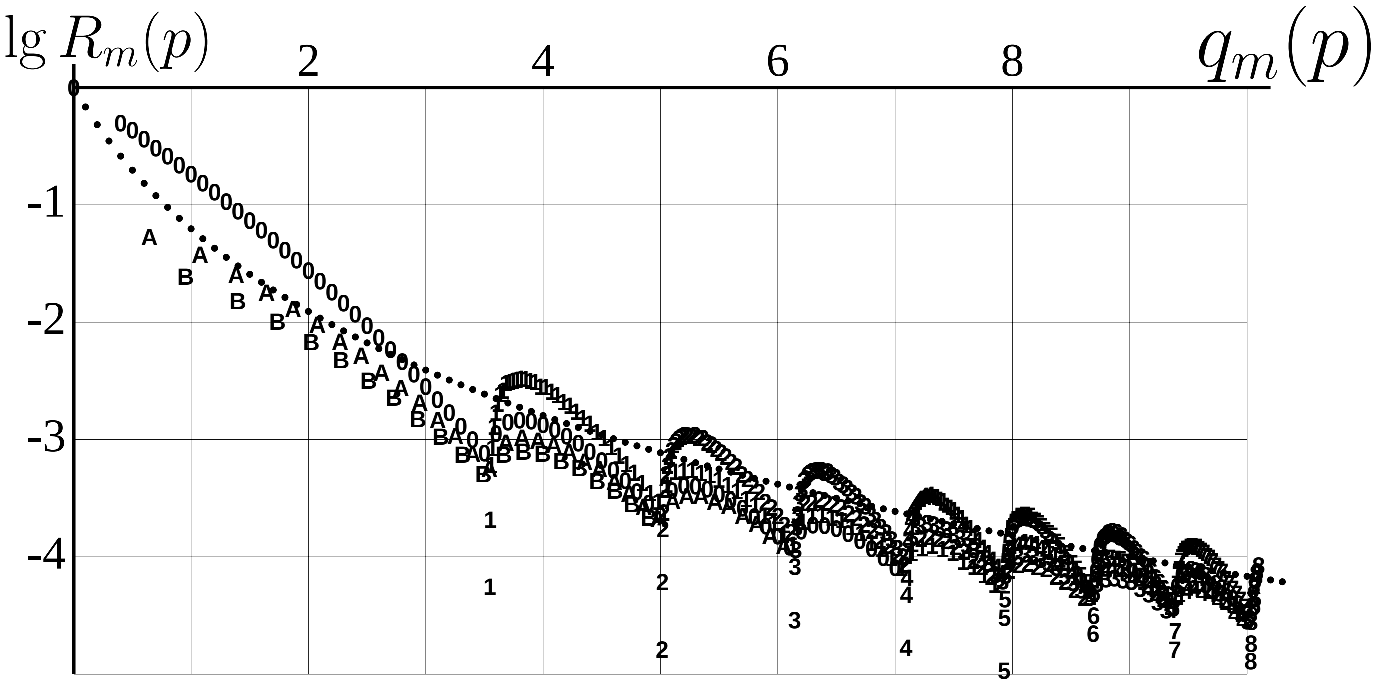

$\lg(R_m(p))$
2
4
6
p
-1
-2
-3
-4
$m=-1$
$m=-2$
$m=1$
$\lg\frac{1}{(1+q_m)^4}$
$\lg\frac{\sqrt{\sqrt{1+1/p^4}+1-\sqrt{2}}}{\sqrt{\sqrt{1+1/p^4}+1+\sqrt{2}}}$
$q_m(p)=\sqrt{4\pi m+p^2}$



$$S_m(p) = R_m(p)\frac{q_m(p)}{p}$$

$$S_0(p) = R_0(p) \approx \exp(-1.68(1+0.018p^2)p \pm 2\%)\ , \quad p<3.6$$

$$\tilde{s}_8(p) = \sum_{m<8} S_m(p) \approx \exp(-1.547(1+0.01p^2)p \pm 2\%)\ , \quad p<3.6$$



Trend: $R_m(p) \approx \frac{1}{\left(1+\sqrt{p^2+4\pi m}\right)^4}$; $q_m(p) = \sqrt{p^2 + 4\pi m}$

Use of symmetry

$$F(X+1,Y) = F(X,Y)\exp(-ip^2/2)$$

$$F(X,Y) = F_{\text{ini}}(X,Y) - \sum_m r_m e^{iq_mY-(i/2)q_m^2X}$$

$$q_m = \sqrt{p_m^2 + 4\pi m}$$

Using the Green funciton (Kirhgof integral):

$$F(1,Y) = \frac{1}{\sqrt{2i\pi}} \int_0^\infty F(0,Y) e^{\frac{i}{2}(Y-Z)^2} \mathrm{d}Z$$

$$e^{-ipY} - \sum_m r_m e^{iq_mY} = \frac{1}{\sqrt{2i\pi}} \int_0^\infty \left(e^{-ipX}\ e^{\frac{i}{2}(Y-Z)^2} - \sum_m r_m e^{\frac{i}{2}(Y-Z)^2} \right) \mathrm{d}Z$$

$$\sum_m \left(e^{iq_mY} - \frac{1}{\sqrt{2i\pi}} \int_0^\infty e^{\frac{i}{2}(Y-Z)^2} \mathrm{d}Z \right) r_m = e^{ipY} - \frac{1}{\sqrt{2i\pi}} \int_0^\infty e^{ipZ} e^{\frac{i}{2}(Y-Z)^2} \mathrm{d}Z$$

$$\sum_m e^{-k_mY} \operatorname{erfc}\left(\frac{Y-ik}{1+i} \right) r_m - e^{-ipY} \operatorname{erfc}\left(\frac{Y+p}{1+i} \right) = 0$$

where $k_m = \sqrt{-4\pi m - p^2}$

P.Sheng, R.S.Stepleman, P.N.Sanda. Exact eigenfunctions for square-wave grating: application to diffraction and surface-plasmon calculations. – Phys. Rev. **B 26** (1982) 2907-2916.

P.C.Logoftu, S.A.Coulombe, B.K.Minhas, J.R.McNeil Identity of the cross-reflection coefficients for symmetric surface-relief grating. – J.Opt.Soc.Am. **A 15**, (1999)p. 1108-1114.

H.Jia, K.Yasumoto. A novel formulation of the Fourier model method in S-matric form for arbitrary chaped gratings. – International Journal of Infrared and Millimeter Waves, **25**, No. 11, (2004) 1591-1609.

Residual $\Phi = \int_0^\infty \left| \sum_m e^{-k_m Y} \mathrm{erfc}\left(\frac{Y-ik}{1+i}\right) r_m - e^{-ipY} \mathrm{erfc}\left(\frac{Y+p}{1+i}\right) \right|^2 \mathrm{d}Y$

can be written as $\Phi = r_m^* A_{m,n} r_n - r_m B_m^* - r_m^* B_m + C$

where

$$
\begin{aligned}
A_{m,n} &= A(k_m^*, k_n) \\
B_m &= A(k_m^*, ip) \\
B_m^* &= A(-ip, k_m) \\
C &= A(-ip, ip)
\end{aligned}
$$

$$A(u,v) = \int_0^\infty e^{-(u+v)x} \, \mathrm{erfc}\left(\frac{x+iu}{1-i}\right) \mathrm{erfc}\left(\frac{x-iv}{1+i}\right) \mathrm{d}x$$

Minimumization of the residual: $\Phi = r^\dagger \, A \, r - r^\dagger \, B - B^\dagger \, r + C$

replace $r \to r + \delta r$

$$\delta\Phi = (\delta r)^\dagger A \, r + r^\dagger A \, \delta r + (\delta r)^\dagger \, B - B^\dagger \, \delta r$$

$$\delta\Phi = (\delta r)^\dagger (A \, r - B) + (A \, r - B)^\dagger (\delta r) \qquad \longrightarrow \qquad Ar = B$$

$$r = A^{-1} B$$

Each choice of number of terms with positive and negative m gives the estimates of reflectivities $|r_m|^2$.

Zero-order estimate: $m = n = 0$

$$A_{0,0} = \int_0^\infty \operatorname{erfc}\left(\frac{x-p}{1-i}\right)\ \operatorname{erfc}\left(\frac{x-p}{1+i}\right)\ \mathrm{d}x$$

$$B_0 = \int_0^\infty e^{-2ip}\ \operatorname{erfc}\left(\frac{x+p}{1-i}\right)\ \operatorname{erfc}\left(\frac{x-p}{1+i}\right)\ \mathrm{d}x$$

$$C = \int_0^\infty \operatorname{erfc}\left(\frac{x+p}{1-i}\right)\ \operatorname{erfc}\left(\frac{x+p}{1+i}\right)\ \mathrm{d}x$$

We get the 0-th order estiamte

$$r_0(p) = \frac{B_0}{A_{0,0}}$$

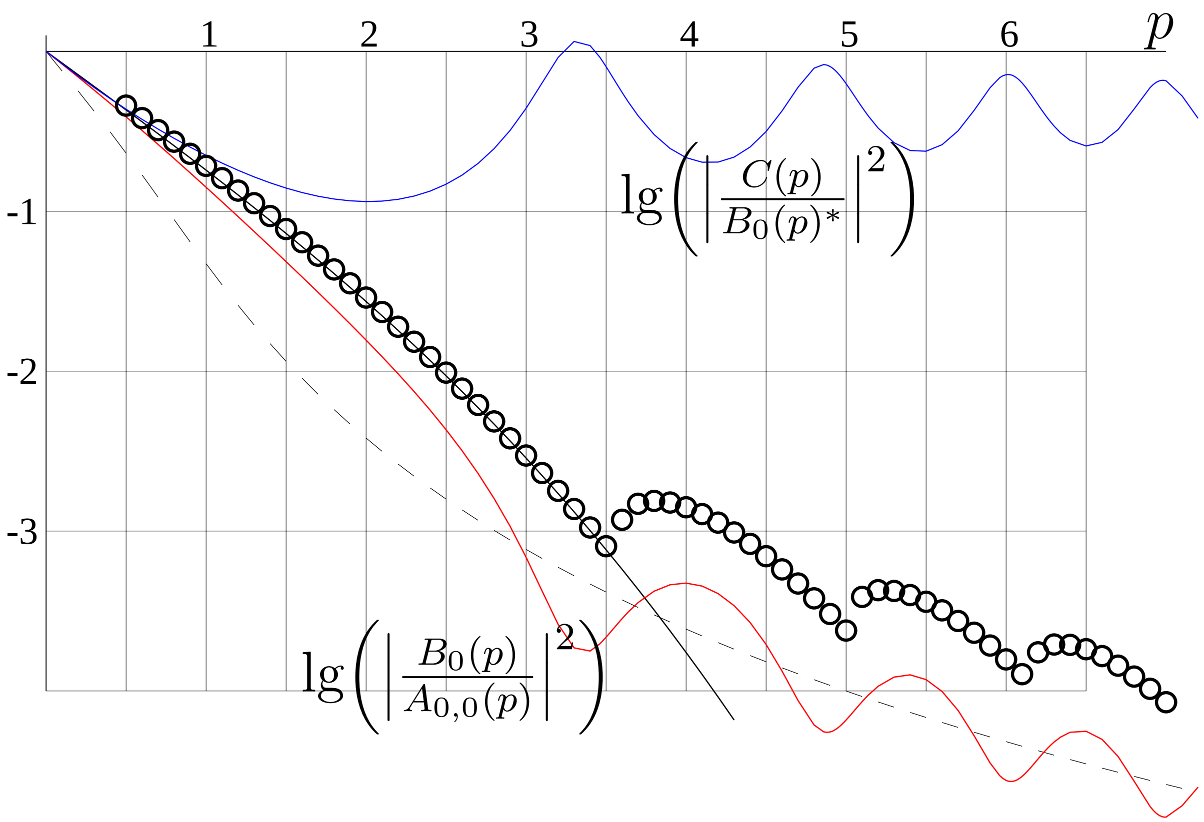
Also, if $\quad \Phi = r^\dagger\ A\ r - r^\dagger\ B - B^\dagger\ r + C \approx 0$

and $\quad Ar = B \quad ,$

then $\quad r \approx C/B^*$

(independent estimate)

Zero order estimate

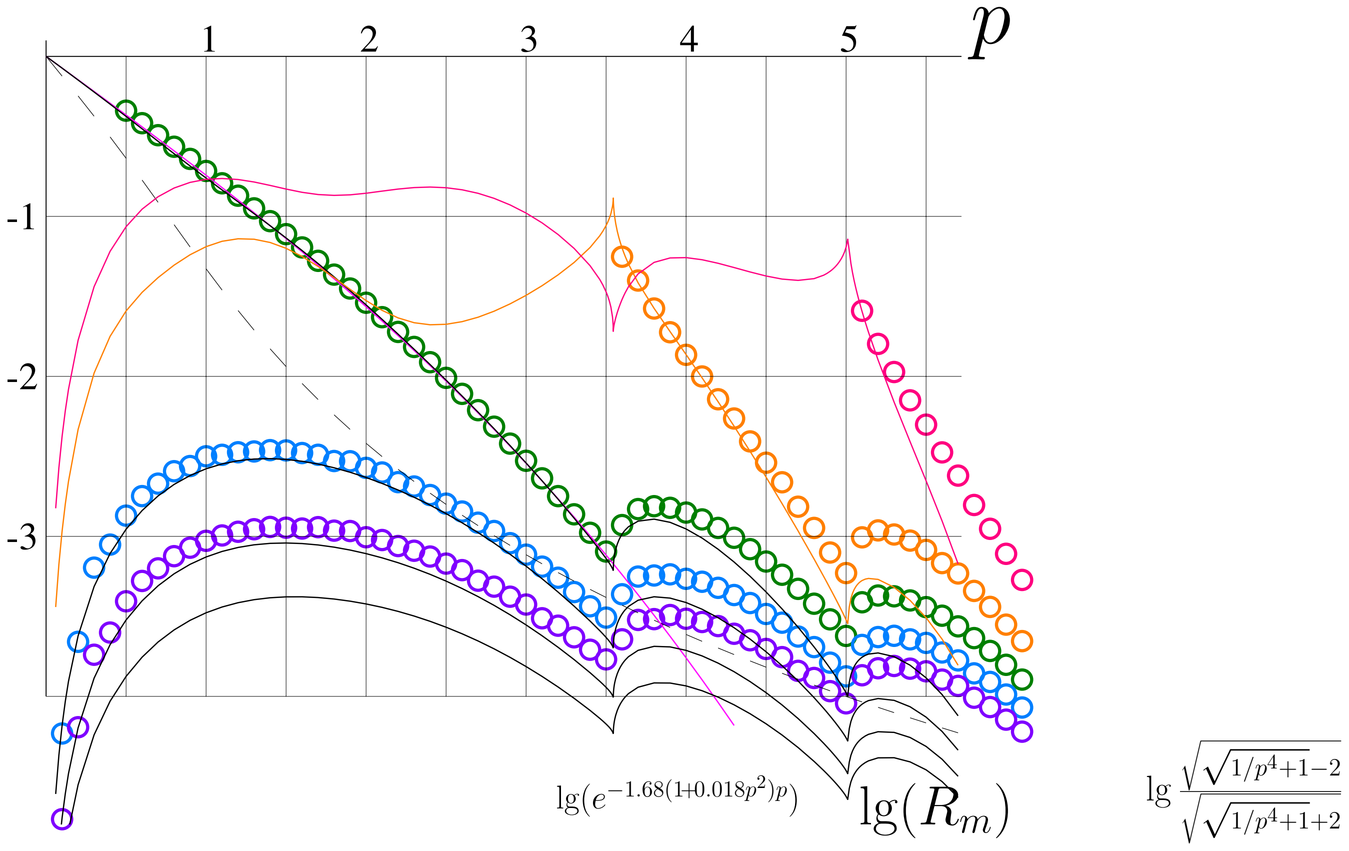


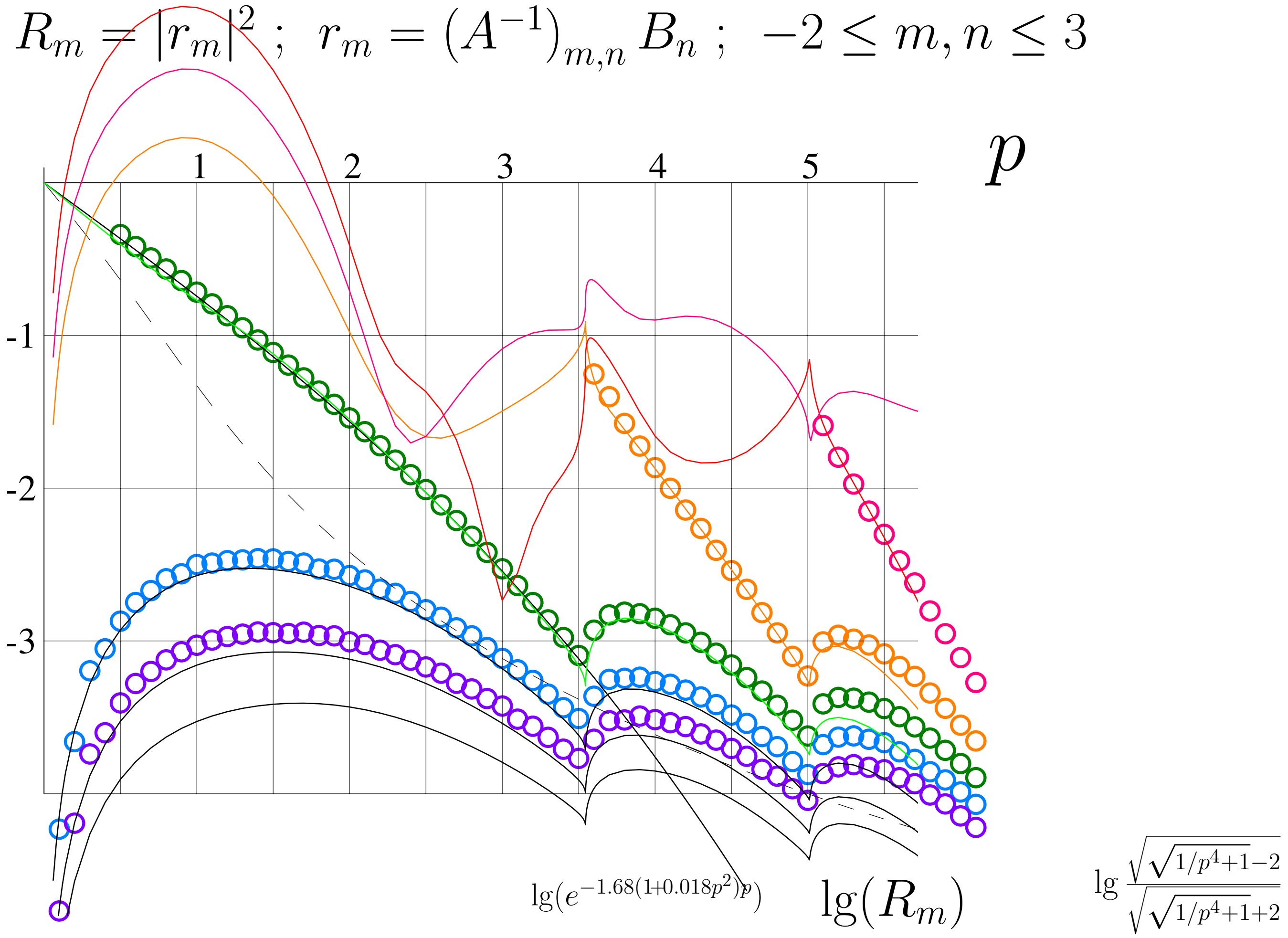
dashed: lg of the Zeno-estimate $R_\mathrm{z}(p) = \frac{\sqrt{\sqrt{1/p^4+1}-2}}{\sqrt{\sqrt{1/p^4+1}+2}}$

Circles: numerical simulations.

black solid: lg of exponential fit $\exp(-1.68(1+0.018p^2)p)$

$$R_m = |r_m|^2 \;;\;\; r_m = \left(A^{-1}\right)_{m,n} B_n \;;\;\; -2 \le m, n \le 3$$





$$R_m = |r_m|^2 \; ; \;\; r_m = (A^{-1})_{m,n} B_n \; ; \;\; -2 \le m, n \le 3$$

Phase of the specular reflection

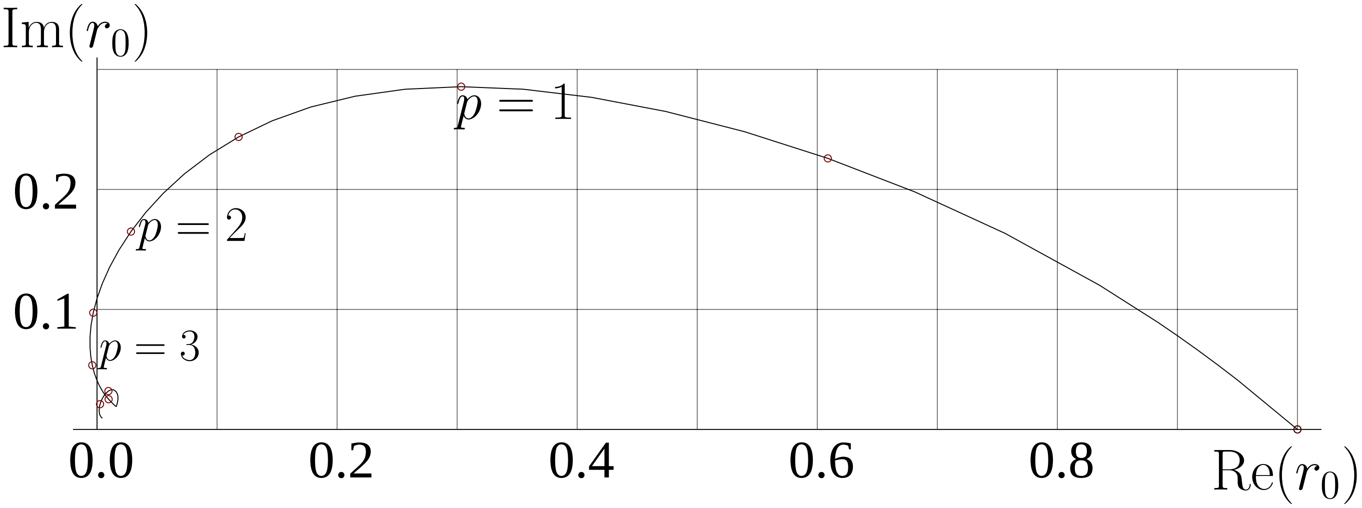
Atomic holograms:

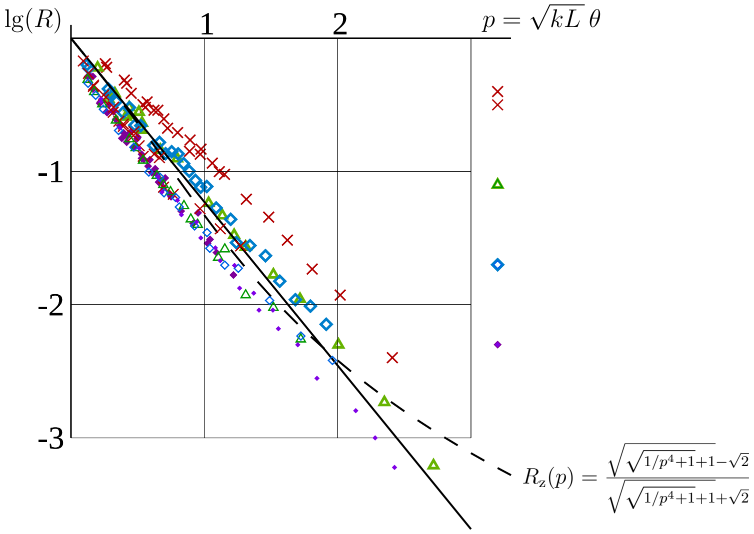
Can we adjust both the amplitude and the phase of the reflected wave?

Varition δh of height of the ridges → $\delta\phi = 2\delta h \sin(\theta)$

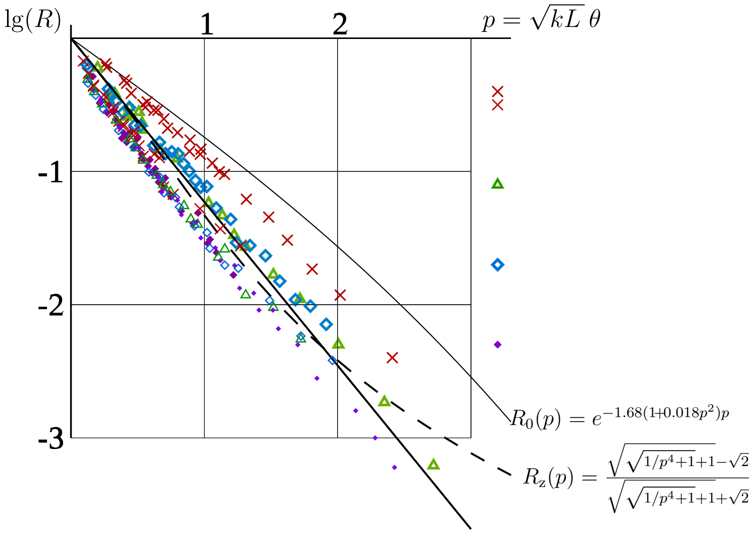
The amplitude can be adjusted with distance L between ridges.

L affects also the phase through $p = \sqrt{kL}\theta$





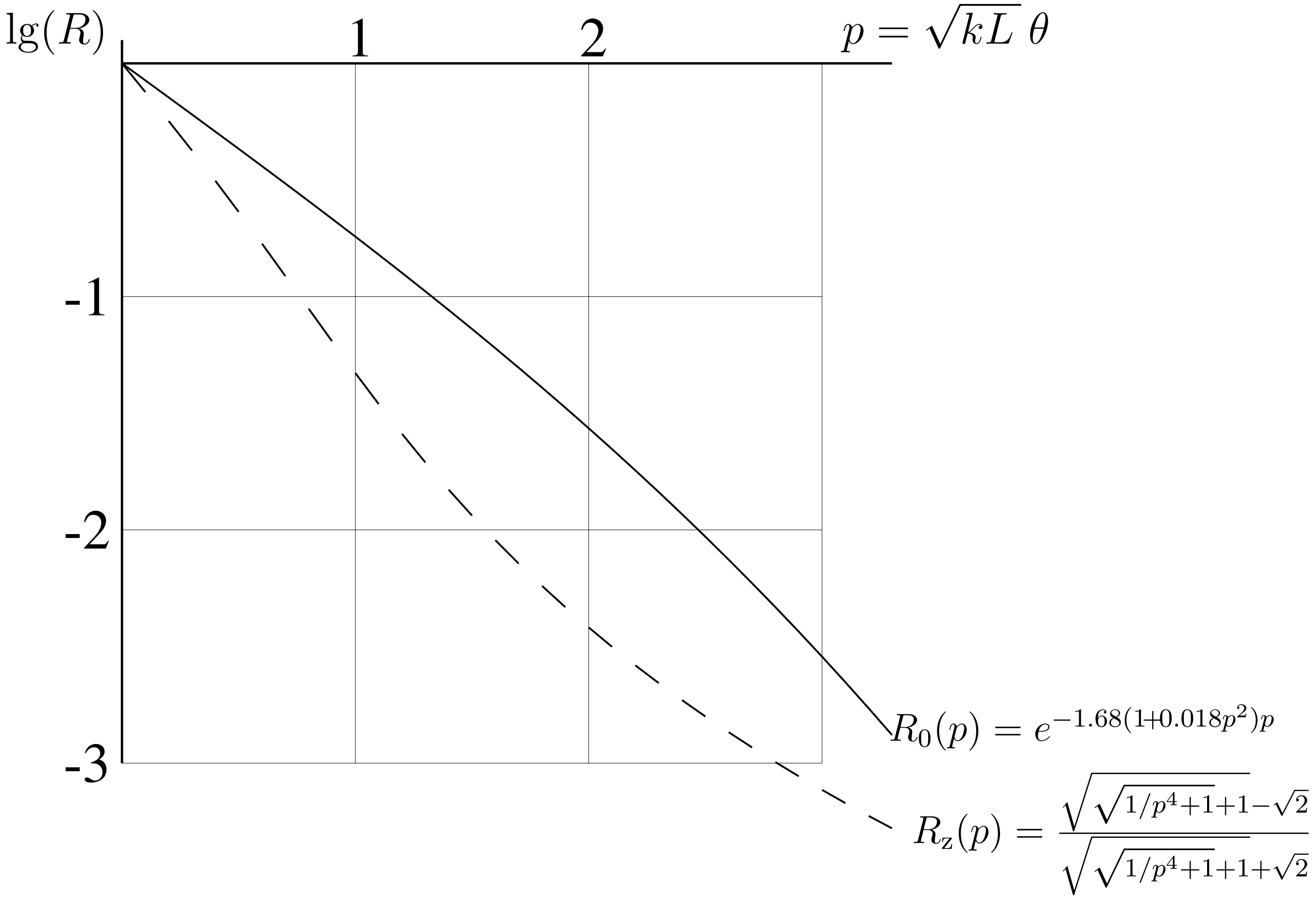
The curves and experimental dots seem to agree.



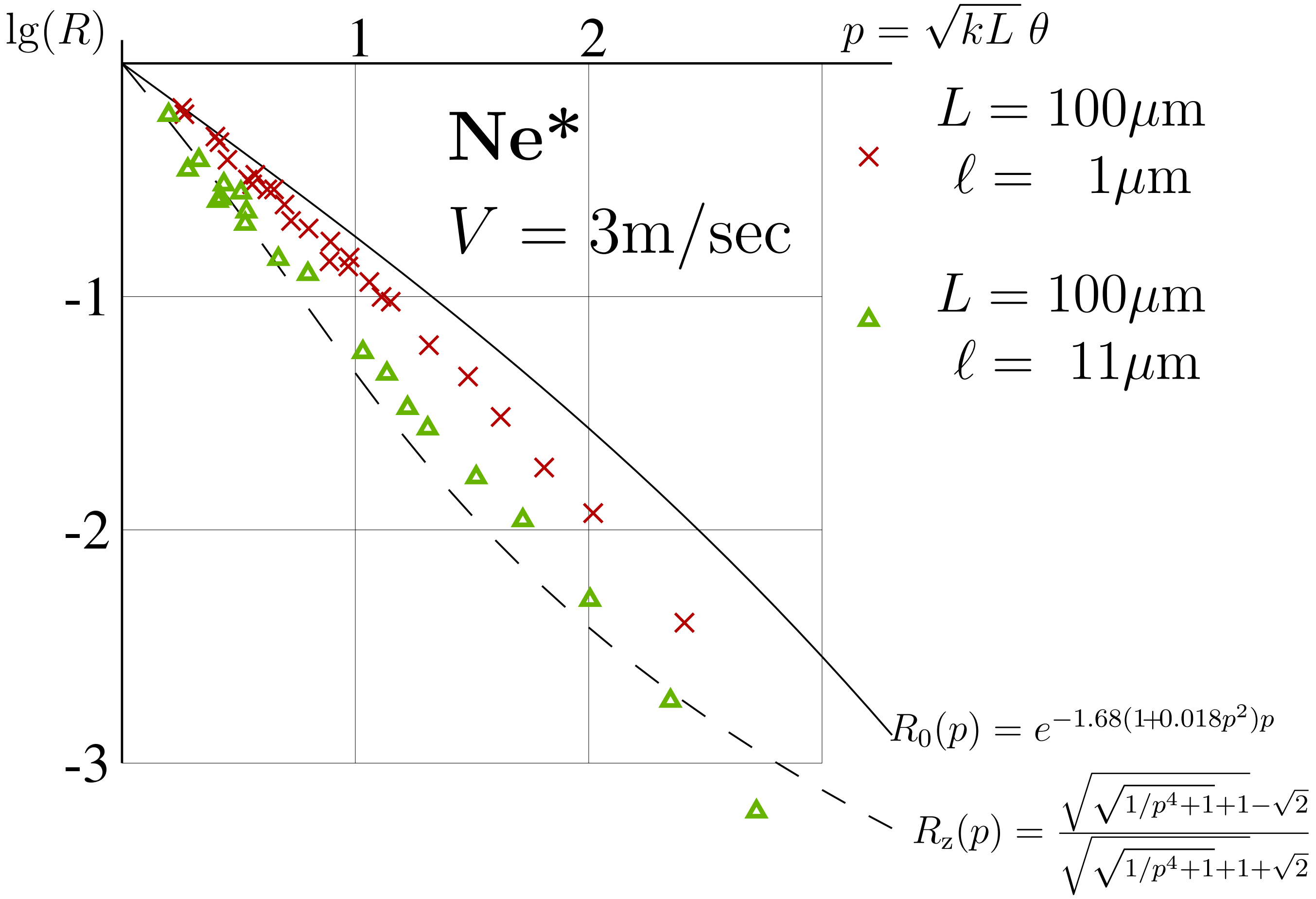
The curves and experimental dots seem to agree.

However, the fit of simulations is a bit above.

What is wrong with simulations?



$\lg(R)$
1
2
$p = \sqrt{kL}\,\theta$
-1
-2
-3
$R_0(p) = e^{-1.68(1+0.018p^2)p}$
$R_z(p) = \frac{\sqrt{\sqrt{1/p^4+1}+1}-\sqrt{2}}{\sqrt{\sqrt{1/p^4+1}+1}+\sqrt{2}}$



$\lg(R)$
1
2
$p = \sqrt{kL}\,\theta$
-1
-2
-3
Ne*
$V = 3\mathrm{m/sec}$
× $L = 100\mu\mathrm{m}$ $\ell = 1\mu\mathrm{m}$
△ $L = 100\mu\mathrm{m}$ $\ell = 11\mu\mathrm{m}$
$R_0(p) = e^{-1.68(1+0.018p^2)p}$
$R_z(p) = \frac{\sqrt{\sqrt{1/p^4+1}+1-\sqrt{2}}}{\sqrt{\sqrt{1/p^4+1}+1+\sqrt{2}}}$

	m	C_4	v	V	ℓ	L
	amu	$10^{-56}\frac{\mathrm{J}}{\mathrm{m}^4}$	m/sec	m/sec	nm	μm
Ne*	20	12	0.019 : 0.374	3	40 : 11000	10 : 100
He*	4	8	0.034 : 1.535	34 : 126	100	5

The Zeno-estimate can be fitted with exponential

$$R_{\mathrm{z}}(p) = \frac{\sqrt{\sqrt{1/p^4+1}+1} - \sqrt{2}}{\sqrt{\sqrt{1/p^4+1}+1} + \sqrt{2}} \approx \exp(-2.8p)$$

At $p<1$, the fit of numerical analysis gives $R_0 \approx \exp(-1.7p)$

Some effect gives a factor $\sim \exp(-1.1\,p)$

Van der Waals potential $U = U(x,y) \approx C_4/y^4$

Can it be responsible for such a factor?

The contribution to the Kirhgoff integral comes from the vicinity of ridges.

Size of this region: $s_0 = \dfrac{1}{k\,\theta}$

This region generates the reflected wave of intensity $R_0\big(\sqrt{kL}\,\theta\big)$

Let $y(x)$ be trajectory. The defasing due to the refraction index becomes strong at

$$\int U(x, y(x)) \frac{\mathrm{d}x}{V} = \hbar$$

$$V = \frac{k\hbar}{m}$$

How close to the ridge should the atom pass in order to feel the ridge?

$$\int \frac{C_4}{s^4} \; \frac{\mathrm{d}x}{V} = \hbar$$

This gives $\dfrac{C_4}{s^4}\;D = \hbar V$ where D is length the atom passes in vicinity of the ridge. For the estimate, we assume $D \approx \ell{+}2s$. Then

$$s = \left(\frac{(\ell + 2s) C_4}{\hbar V}\right)^{1/4}$$

The reflected wave is attenuated proportionally the width s of the "defased" region,

$$R(p,\ell) = \left(1 - \tfrac{s}{s_0}\right) R_0(p)$$

What about the case $s > s_0$? Better, $R(p,\ell) = \exp\!\left(-\tfrac{s}{s_0}\right) R_0(p)$

$$s_0 = \frac{1}{\theta k} = \frac{\hbar}{mV\theta}$$

$$R(p,\ell) = \exp\!\left(-\sqrt{\frac{mV}{\hbar L}}\left(\frac{(\ell\!+\!2s)C_4}{\hbar V}\right)^{1/4} p\right) R_0(p)$$

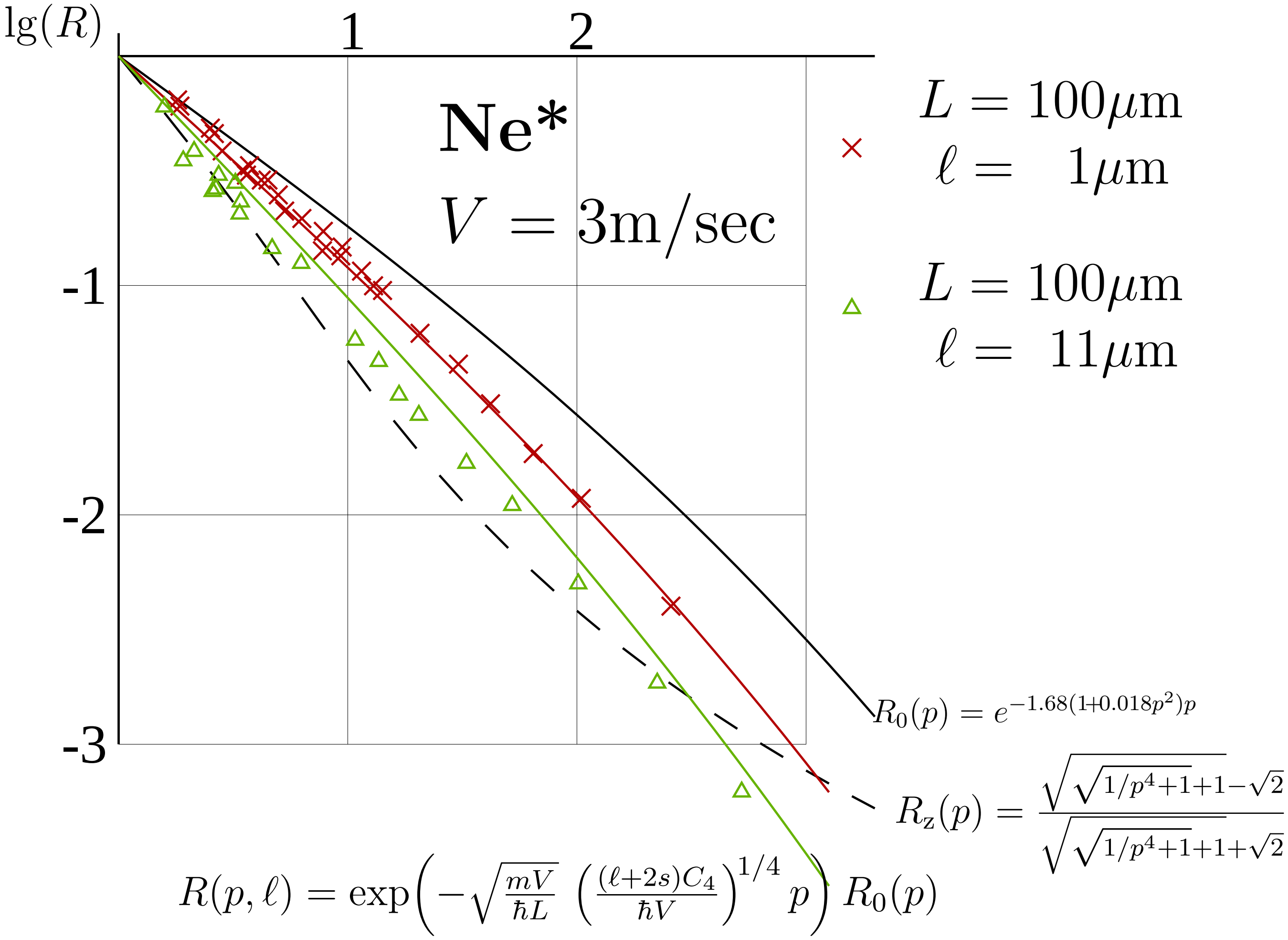
$s = \left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} \approx \left(\frac{(\ell+0)C_4}{\hbar V}\right)^{1/4}$, $p = \sqrt{kL}\theta$, $k = mV/\hbar$

The correction coefficient $\sqrt{\frac{mV}{\hbar L}}\left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} \sim (\ell V)^{1/4}$

has values of order of unity; it is just that we were looking for.

Warning: The speculations above is not a rigorous deguction. It is rather a kitchen which leads to the good fit of experimental data.

The only excuse is the comparizon with experiments.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$R_0(p) = e^{-1.68(1+0.018p^2)p}$
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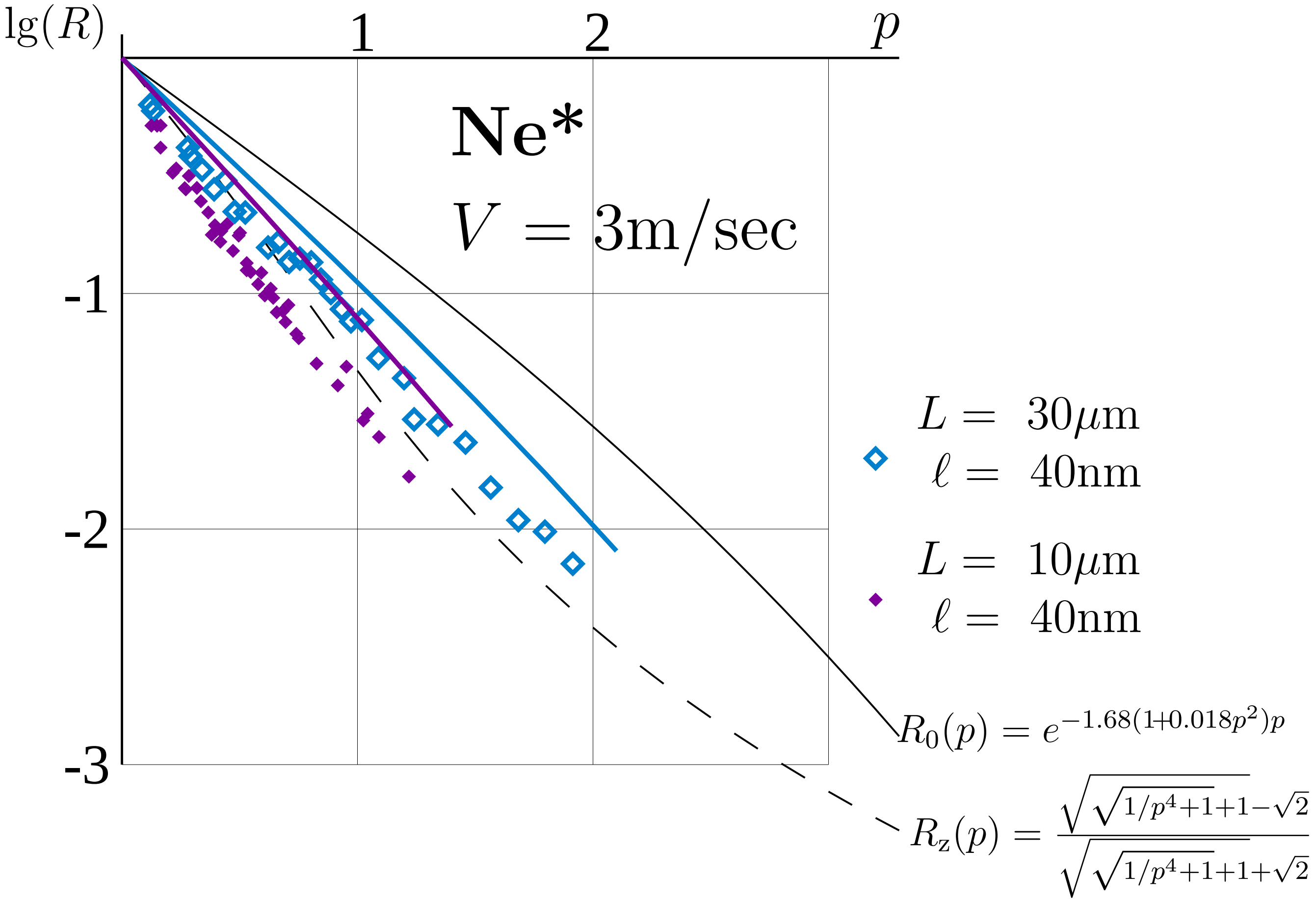
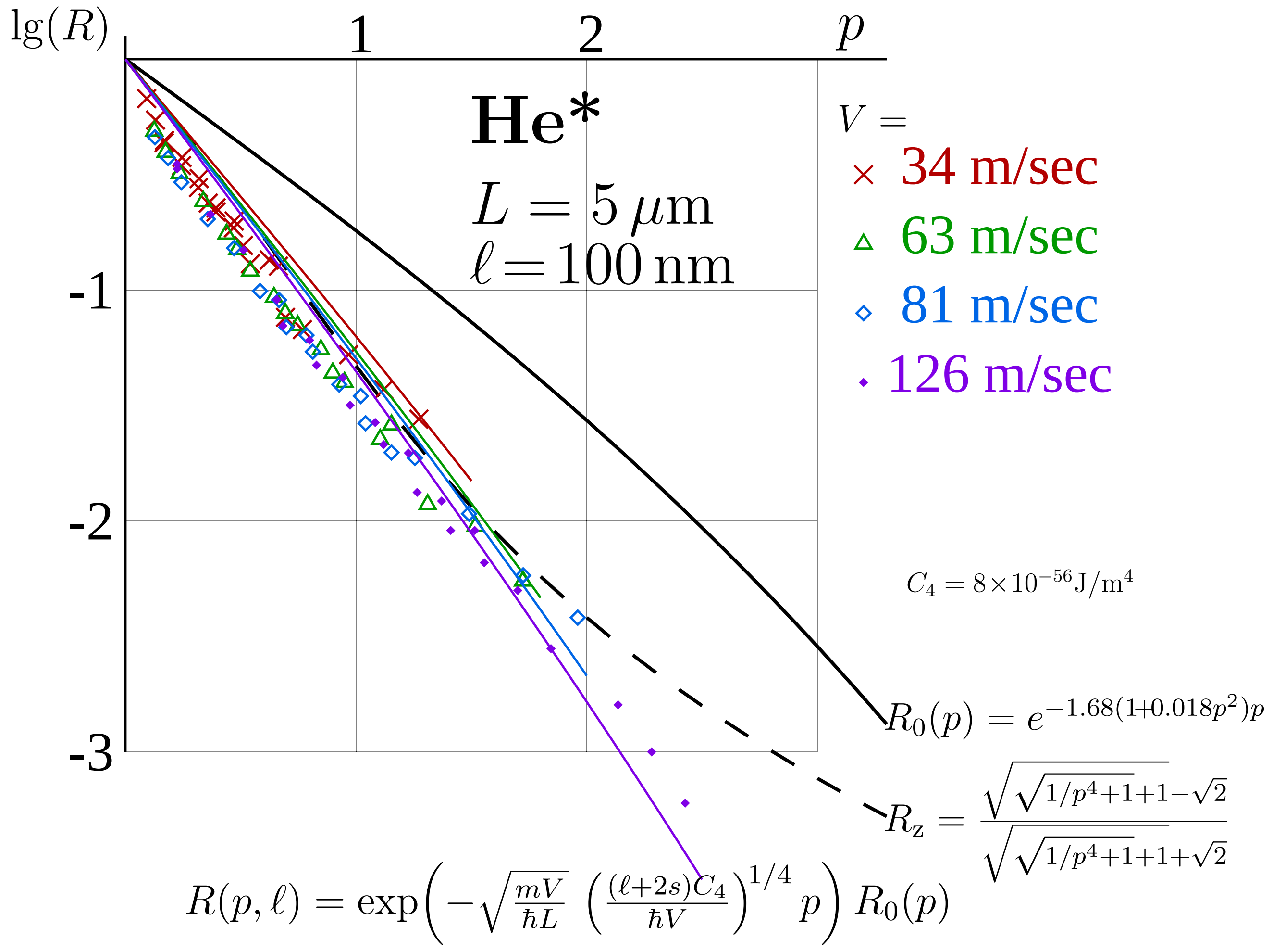
$R(p,\ell) = \exp\left(-\sqrt{\frac{mV}{\hbar L}}\left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} p\right) R_0(p)$



The only two sets of data show significant deviation from the estimate

$R(p,\ell) = \exp\left(-\sqrt{\frac{mV}{\hbar L}}\,\left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} p\right) R_0(p)$


lg(R)
1
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p
-1
-2
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He*
$L = 5\,\mu\mathrm{m}$
$\ell = 100\,\mathrm{nm}$
$V =$
× 34 m/sec
△ 63 m/sec
◇ 81 m/sec
◆ 126 m/sec
$C_4 = 8\times 10^{-56}\,\mathrm{J/m^4}$
$R_0(p) = e^{-1.68(1+0.018p^2)p}$
$R_z = \frac{\sqrt{\sqrt{1/p^4+1}+1-\sqrt{2}}}{\sqrt{\sqrt{1/p^4+1}+1+\sqrt{2}}}$
$R(p,\ell) = \exp\left(-\sqrt{\frac{mV}{\hbar L}}\left(\frac{(\ell+2s)C_4}{\hbar V}\right)^{1/4} p\right) R_0(p)$

CONCLUSIONS

Reflection of atoms from ridged surfaces was treated with optical methods.

Ridged surface as a detector: the continuous detection would give

$$R_{\rm z}=\frac{\sqrt{\sqrt{1/p^4\!+\!1}\!+\!1}\!-\!\sqrt{2}}{\sqrt{\sqrt{1/p^4\!+\!1}\!+\!1}\!+\!\sqrt{2}} \quad , \qquad p=\sqrt{kL}\,\theta$$

The more detailed description of narrow risges leads to the empiric fit

$$R_0(p)\approx \exp(-1.68(1\!+\!0.018p^2)p\pm 2\%)~, \quad p\!<\!3.6$$

Correction due to the van der Waals interaction:

$$R(p,\ell)=\exp\!\left(-\sqrt{\frac{mV}{\hbar L}}\,\left(\frac{(\ell+2s)\,C_4}{\hbar V}\right)^{1/4} p\right) R_0(p)$$

The estimates $R_z(p)$ and $R(p,\ell)$ show good agreement with experiments.

The most of formulas of this paper are not specific for atomic waves; the estimates should work as follows for waves of any other origin (photons, neutrons, phonons; even oceanic waves at the surface of water). The analysis suggested can be used in the design of ridged mirrors for warious applications including atomic optics.

Thank Jun-ichi Fujita for his beautiful samples.
Thank Hilmar Oberst, Fujio Shimizu and Kazuko Shimizu for the original data.
Thanks J.-F. Bisson, K. Nakagawa, M. Morinaga and K. Ueda for valuable discussions.
Thank the Japan Society for the Promotion of Science.
This work was partly supported by the COE project “Coherent Optical Science” of the Ministry of Education, Culture, Sports, Science and Technology, Japan.

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