Broadband laser materials and the McCumber relation

Dmitrii Kouznetsov. Inst. for Laser Science, Univ. of Electro-Communications, Japan

`http://dima@ils.uec.ac.jp~dima`

Concept of the emission cross-section.

Emission spectra for $Yb:Gd_2SiO_5$ published recently: their interpretation as emission cross-section is not correct.

Gedanken experiments: use of emission cross-section implies the Mc-Cumber relation.

Any deviation from the Mc-Cumber relation allows the heat transfer from cold object to the hot one.

The McCumber relation has the same range of validity as the concept of the emission cross-section.

The correction of the published emission cross-sections on the base of absorption cross-section.

keywords: McCumber relation, emission cross-section, ytterbium-doped oxyorthosilicates

W.Li, H.Pan, L.Ding, H.Zeng, W.Lu, G.Zhao, C.Yan, L.Su, J. Xu. **Efficient diode-pumped Yb: Gd_2SiO_5 laser**. – Appl. Phys. Lett. **88**, 221117 (2006)

W.Li, H.Pan, L.Ding, H.Zeng, G.Zhao, C.Yan, L.Su, J.Xu. **Diode-pumped continuous-wave and passively mode-locked Yb : Gd_2SiO_5 laser**. – Optics Express **14**, 686-695 (2006)

C.Yan, G.Zhao, L.Zhang, J.Xu, X.Liang, D.Juan, W.Li, H.Pan, L.Ding, H.Zeng. **A new Yb-doped oxyorthosilicate laser crystal: Yb : Gd_2SiO_5** .
– Solid State Comm.**137**, 451-455 (2006)

D.E.McCumber, **Einstein relations connecting broadband emission and absorption spectra**. – Phys.Rev. **136** (4A), 954-957 (1964)

P.C.Becker, N.A.Olson, J.R.Simpson. **Erbium-doped fiber amplifiers: fundamentals and theory**. – Academic, 1999.

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A. Kaminsky. Private communication, 2006.

A. Shirakawa. Private communication, 2006.

Editors, reviewers and other colleagues:

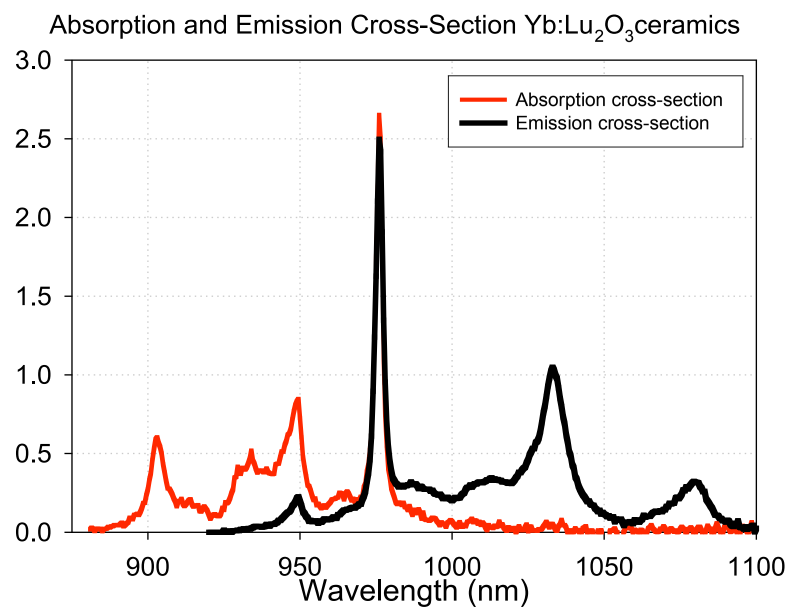
Emission cross-section of a material with different sites has no need to satisfy the McCumber relations!

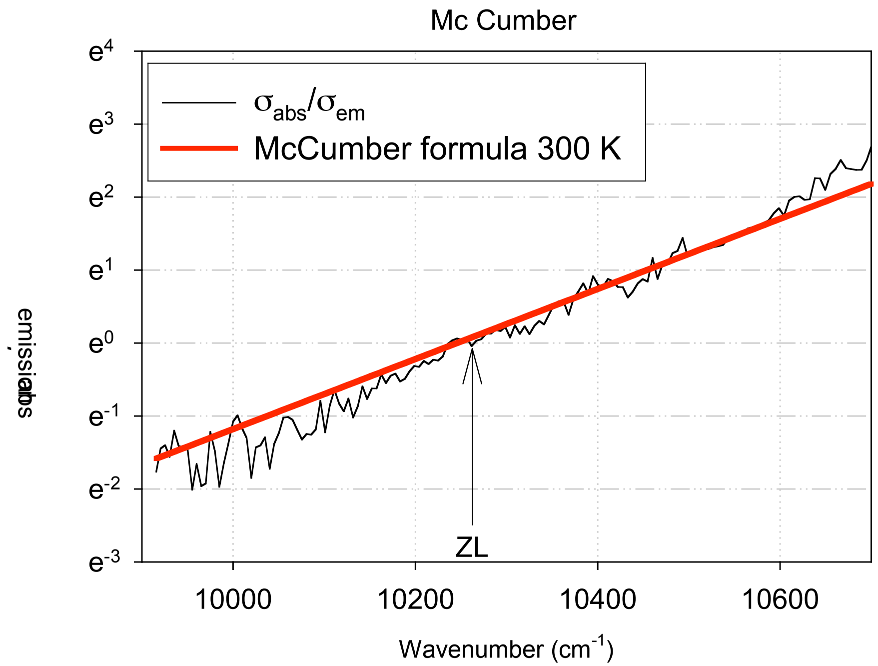
?

Example of a check of experimental data for the McCumber relation

Data for $3\%\mathrm{Yb} : \mathrm{Lu_2O_3}$
by J.-F.Bisson et al., Optics Japan 2006, Tokyo

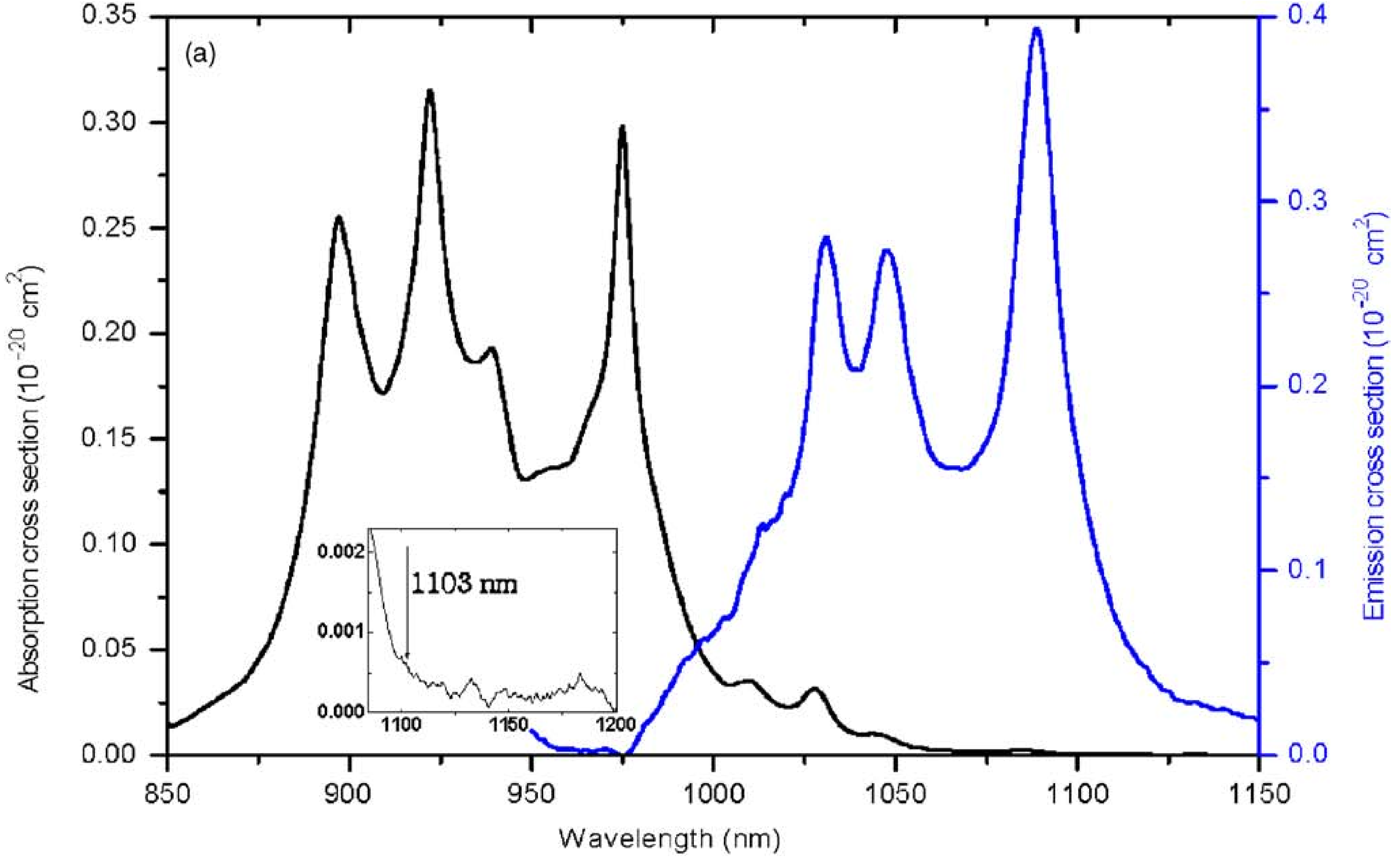
$$\frac{\sigma_{\mathrm{a}}(\lambda)}{\sigma_{\mathrm{e}}(\lambda)} = \exp\left[\frac{hc}{k_B T}\left(\frac{1}{\lambda} - \frac{1}{\lambda_{\mathrm{ZL}}}\right)\right]$$





MATERIAL FOR PERPETUAL MOTION

W.Li, H.Pan, L.Ding, H.Zeng, W.Lu, G.Zhao, C.Yan, L.Su, J. Xu.
Efficient diode-pumped Yb: Gd_2SiO_5 laser
Appl. Phys. Lett. **88**, 221117 (2006)



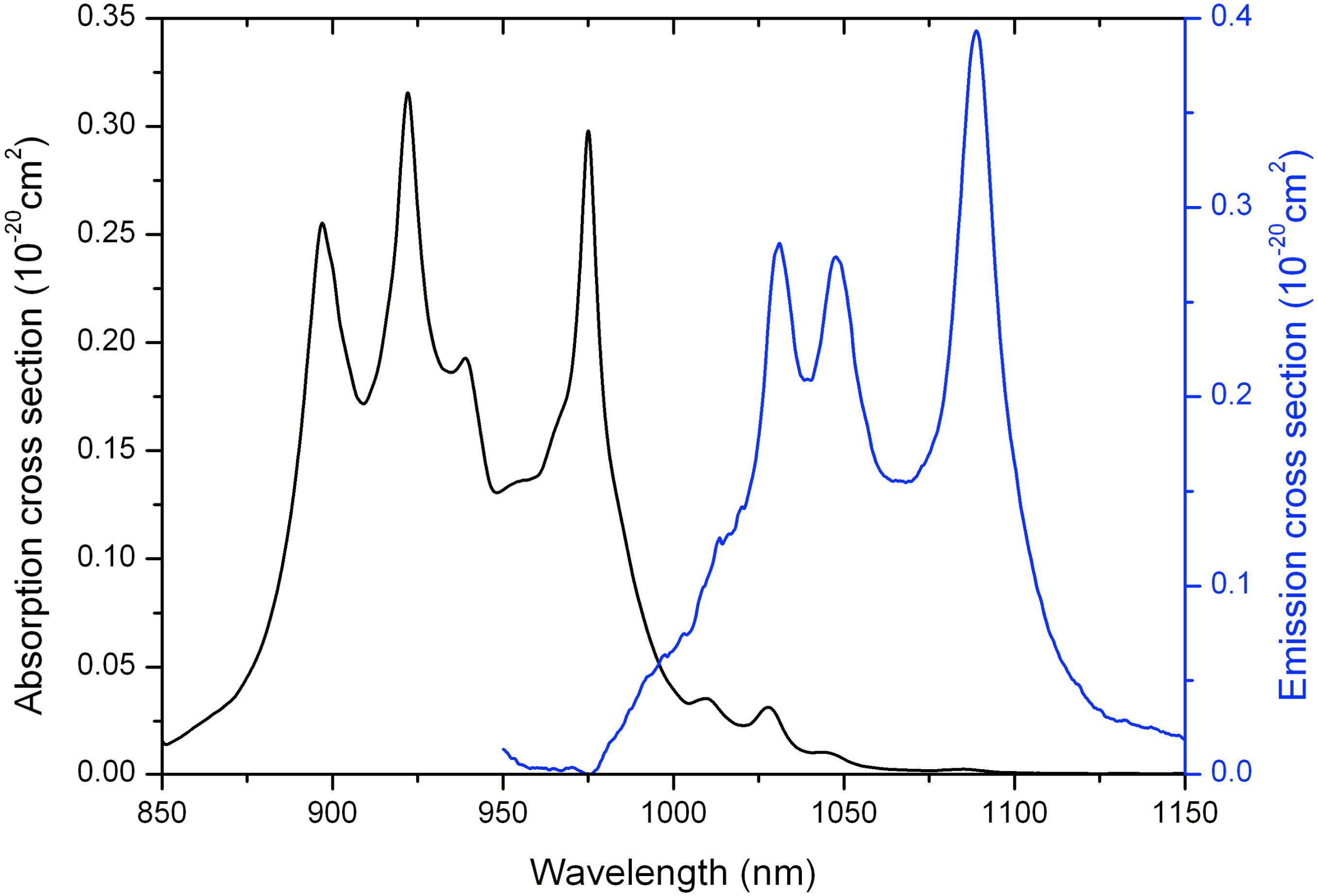


Fig. 1. Room-temperature absorption and emission spectra of the Yb:GSO laser crystal.

#9000 - $15.00 USD
Received 3 October 2005; revised 5 December 2005; accepted 5 December 2005

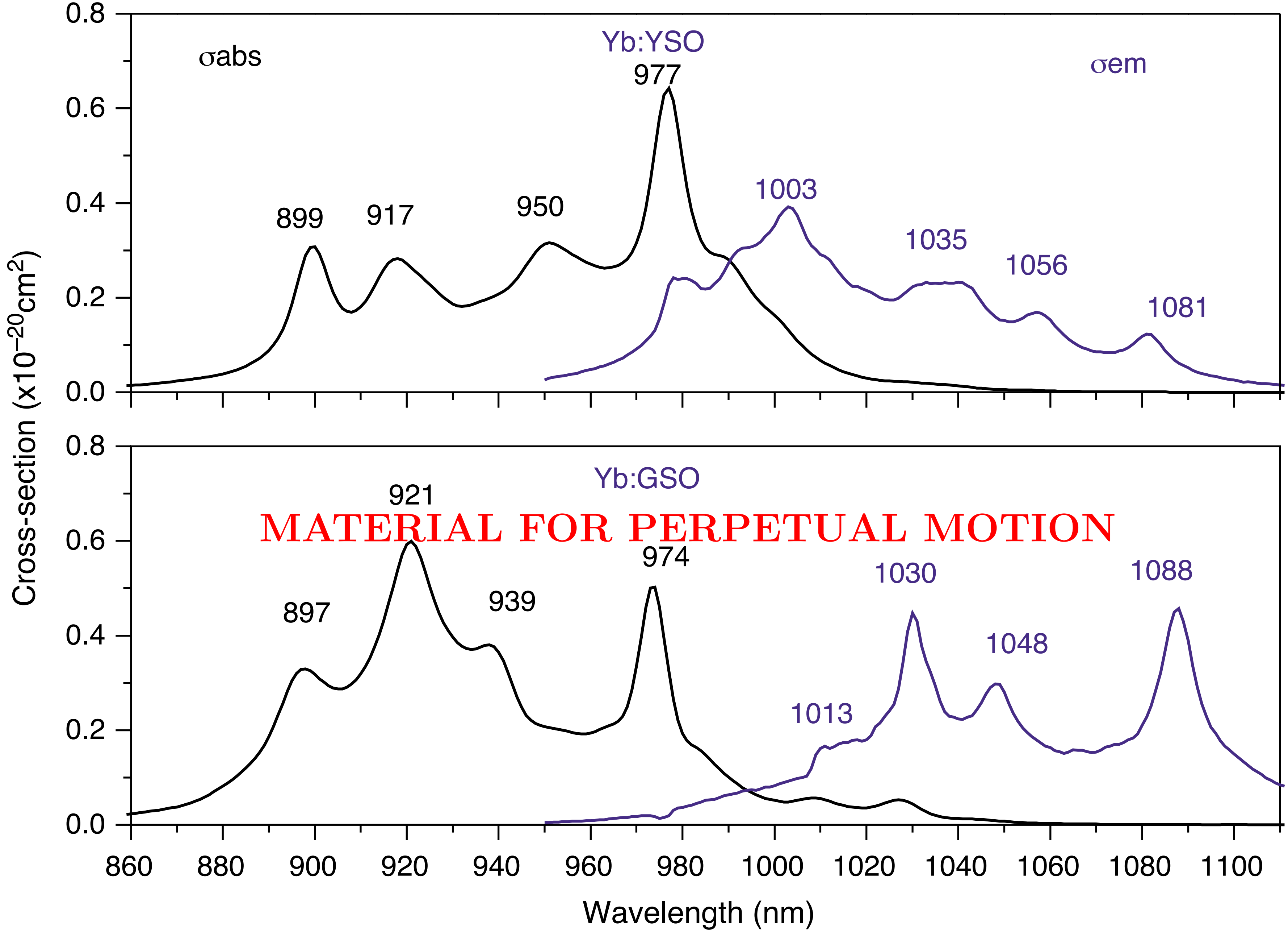
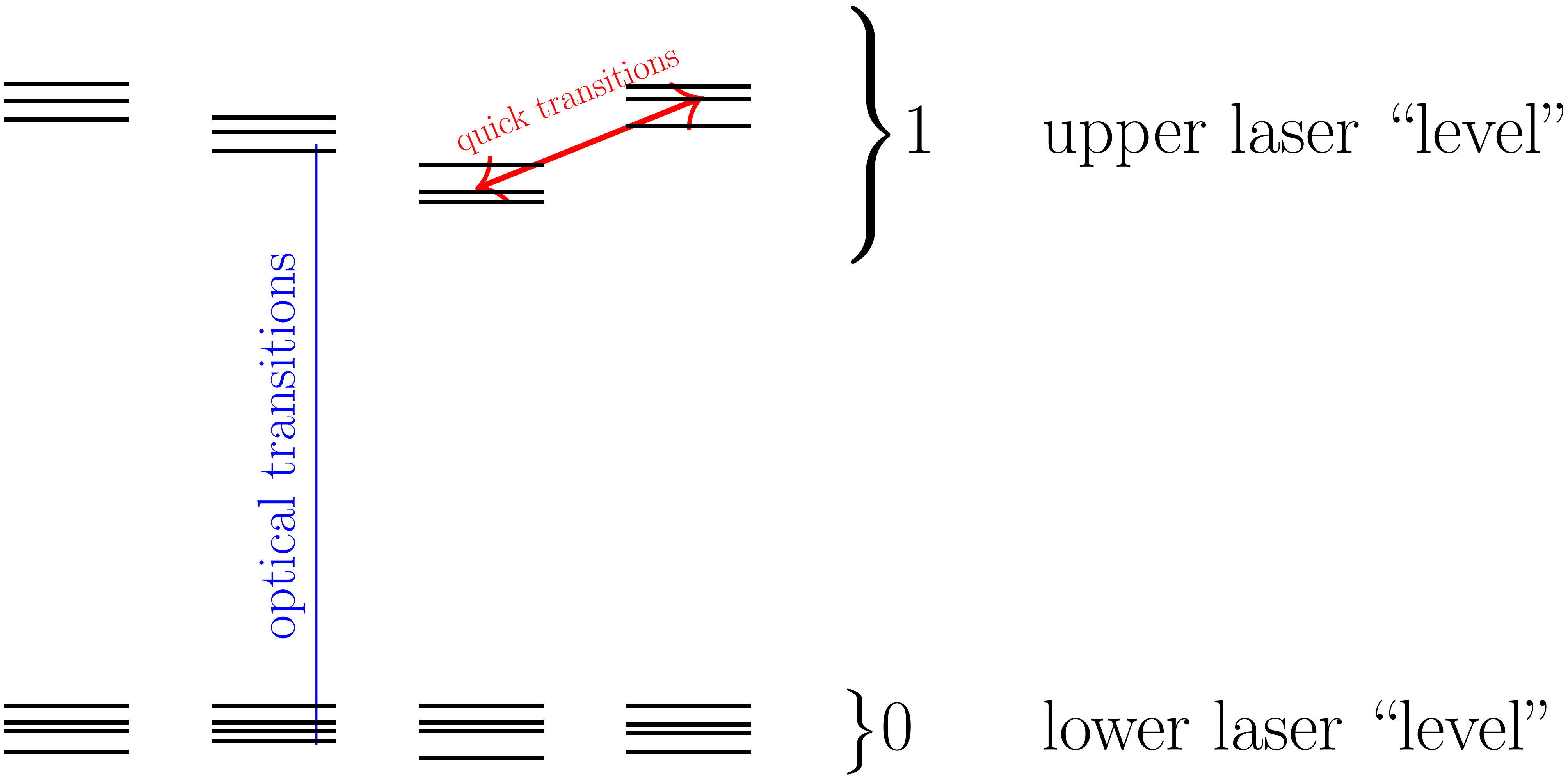


Fig. 3. The absorption cross section σ_{abs} and the emission cross section σ_{em} of 5 at.% Yb:GSO and 5 at.% Yb:YSO.

What is **Emission cross-section** ?

gain: $$g(\omega) = n_1\,\sigma_{\mathrm{e}}(\omega) - n_0\,\sigma_{\mathrm{a}}(\omega)$$



McCumber relation

$$\sigma_{\mathrm{e}}(\omega) = \sigma_{\mathrm{a}}(\omega)\exp\left(\frac{\hbar\omega_{\mathrm{z}}}{K_{\mathrm{B}}T} - \frac{\hbar\omega}{K_{\mathrm{B}}T}\right)$$

Zero-line: $\sigma_{\mathrm{a}}(\omega_{\mathrm{z}}) = \sigma_{\mathrm{e}}(\omega_{\mathrm{z}})$

The McCumber relation holds well for various Er, Yb and Nd doped materials

What about $\mathbf{Yb : Gd_2SiO_5}$?

Deduction of the McCumber relation

Let N_m be number of photons in the m-th mode of an idealized box cavity;

$$\frac{\mathrm{d}N_m}{\mathrm{d}t} = (N_m + 1)\, n_1\, v(\omega_m)\, \sigma_{\mathrm{e}} - N_m\; n_0\, v(\omega_m)\, \sigma_{\mathrm{a}} \quad ,$$

ω_m is frequency of the mth mode ; $v(\omega)$ is group velocity

conservation of number of excitations: $V n_1 + \sum_m N_m = \mathrm{constant}$

In the steady-state: $(N_m + 1)\, n_1\, v(\omega_m)\, \sigma_{\mathrm{e}} = N_m\; n_0\, v(\omega_m)\, \sigma_{\mathrm{a}}$

Bose distribution: $N_m = \frac{1}{\mathrm{e}^{\hbar\omega m/(k_{\mathrm{B}}T)} - 1}$

then

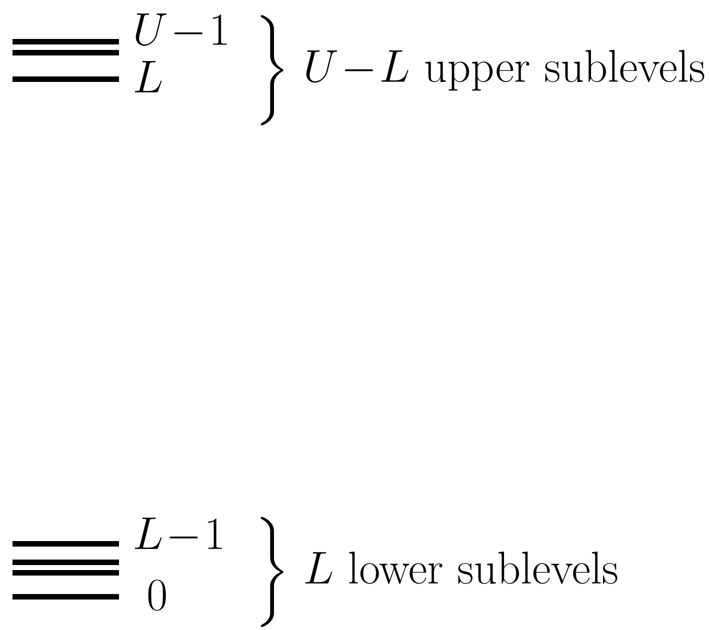
$$\frac{\sigma_{\mathrm{a}}(\omega)}{\sigma_{\mathrm{e}}(\omega)} = \left(\frac{n_1}{n_0}\right)_T \exp\left(\frac{\hbar\omega}{k_{\mathrm{B}}T}\right)$$

The McCumber relation is not a specific property of homogeneously-broadened media.
It holds for any medium where the effective emission cross-section has sense.

Zero-line frequency ω_z

$$\left(\frac{n_1}{n_0}\right)_T = \exp\left(-\frac{\hbar\omega_\mathrm{z}}{k_\mathrm{B}T}\right)$$

Numeration of sublevels:



Case of homogeneous medium:

$$\left(\frac{n_1}{n_0}\right)_T = \frac{\sum_{i=L}^{U-1} \exp\left(-\frac{\hbar\varepsilon_i}{K_\mathrm{B}T}\right)}{\sum_{i=0}^{L-1} \exp\left(-\frac{\hbar\varepsilon_i}{K_\mathrm{B}T}\right)} \approx \exp\left(-\frac{\hbar\varepsilon_L}{K_\mathrm{B}T}\right)$$

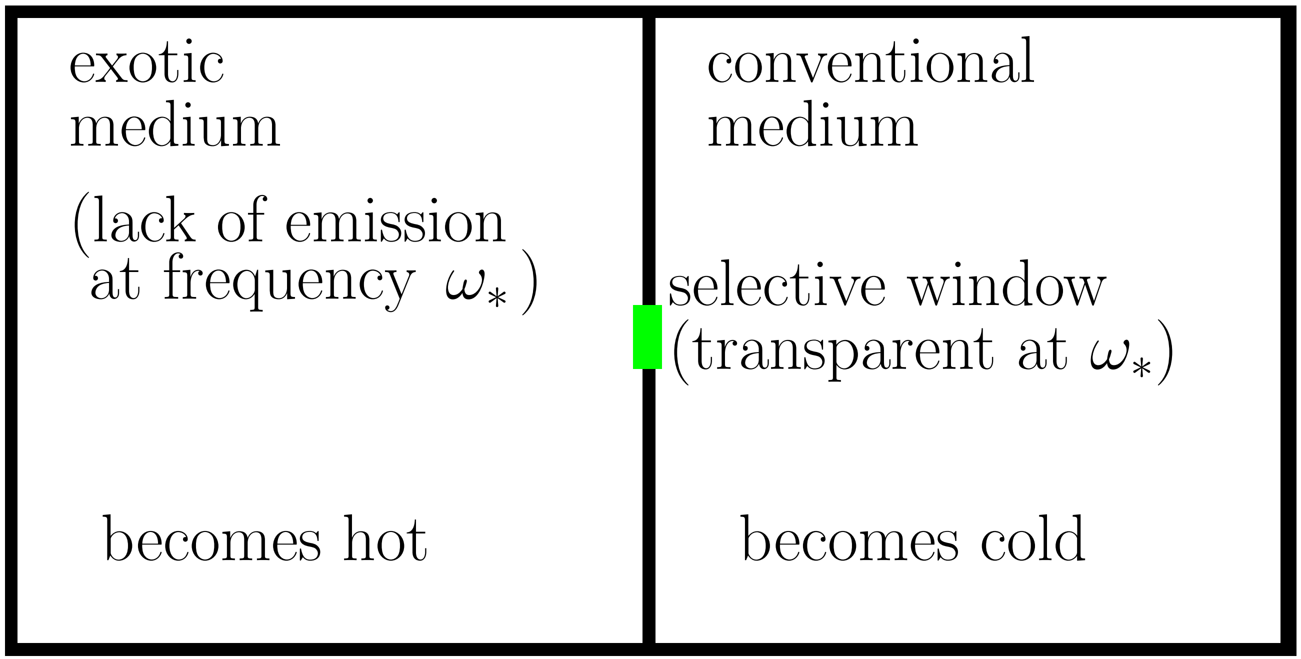
Case of a medium with different active sites; let q_s be concentration of sth site:

$$\left(\frac{n_1}{n_0}\right)_T = \frac{\displaystyle\sum_s q_s \frac{\sum_{i=L}^{U-1} \exp\left(-\frac{\hbar\varepsilon_{s,i}}{K_\mathrm{B}T}\right)}{\sum_{i=0}^{U-1} \exp\left(-\frac{\hbar\varepsilon_{s,i}}{K_\mathrm{B}T}\right)}}{\displaystyle\sum_s q_s \frac{\sum_{i=0}^{L-1} \exp\left(-\frac{\hbar\varepsilon_{s,i}}{K_\mathrm{B}T}\right)}{\sum_{i=0}^{U-1} \exp\left(-\frac{\hbar\varepsilon_{s,i}}{K_\mathrm{B}T}\right)}}$$

$$\omega_\mathrm{z} = -\frac{k_\mathrm{B}T}{\hbar} \ln\left(\frac{n_1}{n_0}\right)_T$$

Gedanken experiment: perpetual motion

Assume that for some exotic medium, the emission and absorption cross-sections deviate from McCumber (at least in some narrow interval of frequencies)

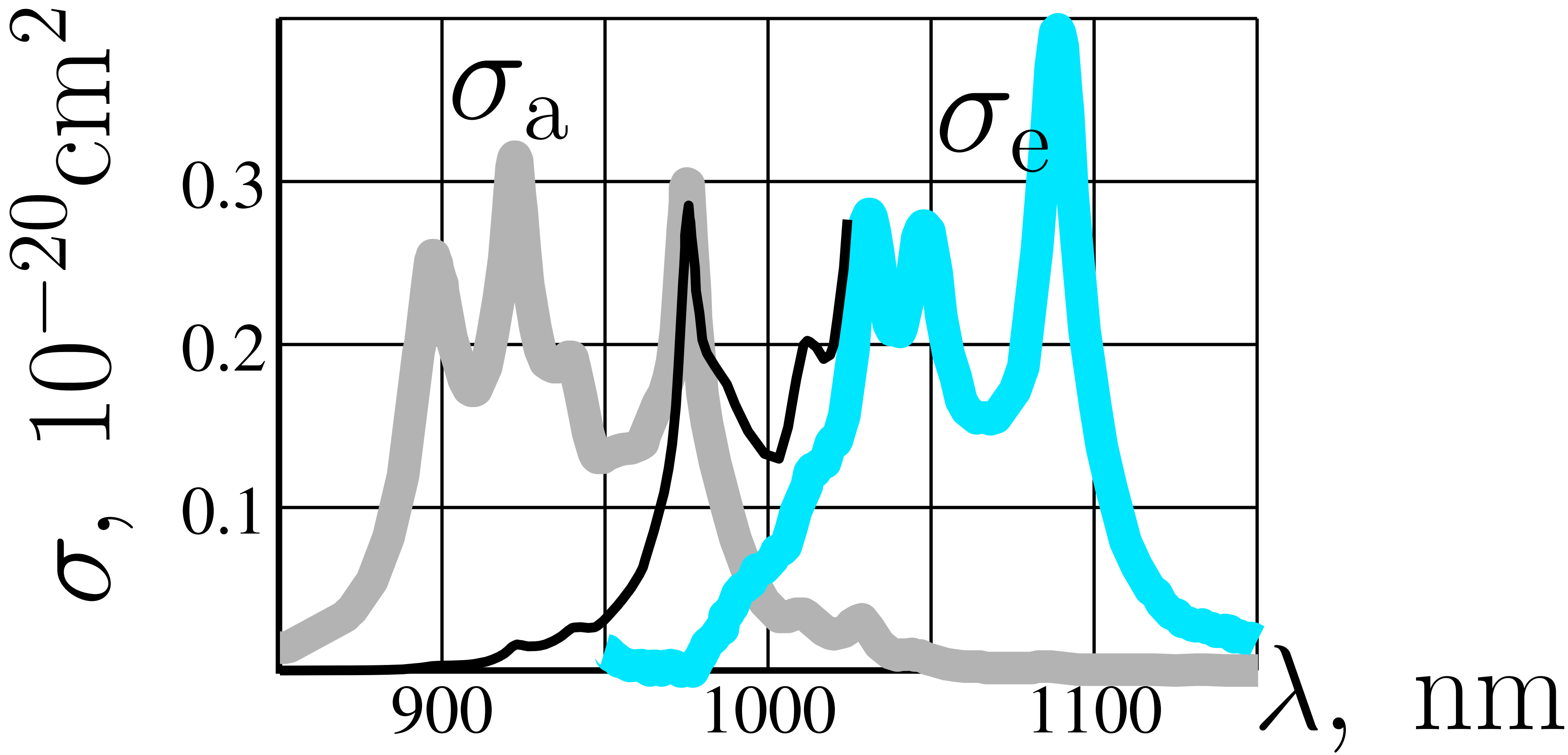


If for some medium, in some approximation, both emission cross-section and absorption cross-section can be defined,

then either these cross-section satisfy the McCumber

or the medium allows the perperual motion of Second kind.

Correction of σ_e for $Yb:Gd_2SiO_5$



Thick curves: σ by App.Ph.Let.**88**,221117(2006) and Opt.Express **14**,686-695(2006)

Thin black curve: σ_e, reconstructed with McCumber from σ_a.

Values used for reconstruction: $T = (271 + 14)$ Kelvin , $\omega_z = \dfrac{2\pi c}{975 \text{ nm}}$

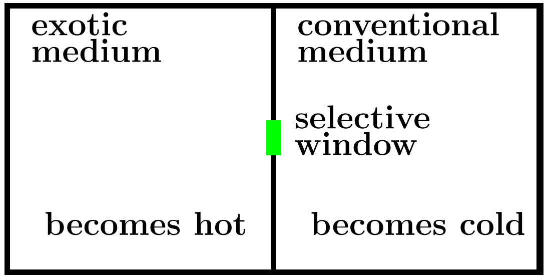
Conclusions

$$\frac{\sigma_{\rm a}(\omega)}{\sigma_{\rm e}(\omega)} = \left(\frac{n_1}{n_0}\right)_T \exp\left(\frac{\hbar\omega}{k_{\rm B}T}\right)$$

McCumber relation applies not only to homogeneous laser media, but to any medium with fast transitions within groups of sub-levels.

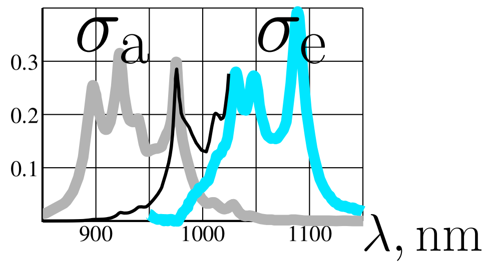
If the concept of emission cross-section has sense for some medium, then, for this medium, the McCumber holds.

Any medium with effective cross-sections, violating the McCumber relation, allows the perpetual motion of Second kind.



Spectra of $Yb:Gd_2SiO_5$ by Appl.Phys.Lett and Optics Express cannot be interpreted as emission cross-section. Such interpretation contradicts the Second Law.

If $\sigma_{\rm a}$ of $Yb:Gd_2SiO_5$ was measured correctly, then $\sigma_{\rm e}$ can be reconstructed with McCumber.



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