

Broadband laser materials and the McCumber relation

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Concept of the emission cross-section.

Emission spectra for Yb : Gd₂ Si O₅ published recently: their interpretation as emission cross-section is not correct.

Gedanken experiments: use of emission cross-section implies the McCumber relation.

Any deviation from the McCumber relation allows the heat transfer from cold object to the hot one.

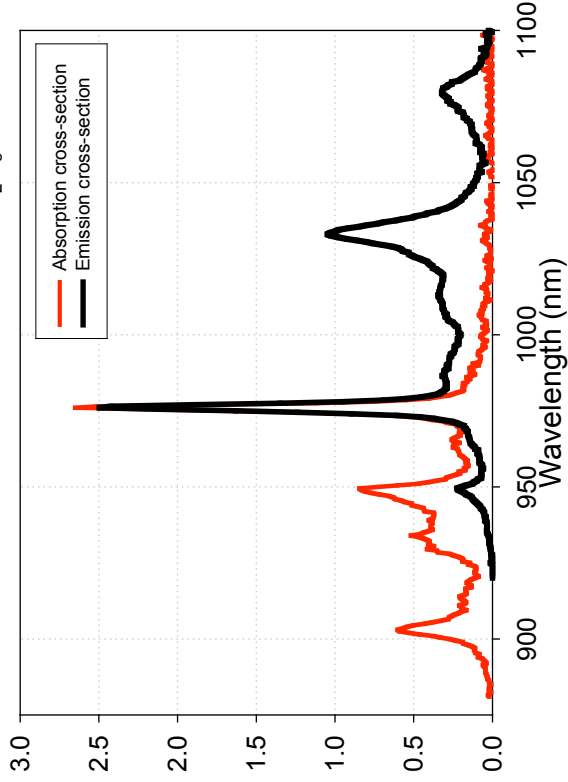
The McCumber relation has the same range of validity as the concept of the emission cross-section.

The correction of the published emission cross-sections on the base of absorption cross-section.

keywords: McCumber relation, emission cross-section, ytterbium-doped oxyorthosilicates

- W.Li, H.Pan, L.Ding, H.Zeng, W.Lu, G.Zhao, C.Yan, L.Su, J. Xu. **Efficient diode-pumped Yb : Gd₂ Si O₅ laser**. – Appl. Phys. Lett. **88**, 221117 (2006)
- W.Li, H.Pan, L.Ding, H.Zeng, G.Zhao, C.Yan, L.Su, J.Xu. **Diode-pumped continuous-wave and passively mode-locked Yb : Gd₂SiO₅ laser**. – Optics Express **14**, 686-695 (2006)
- C.Yan, G.Zhao, L.Zhang, J.Xu, X.Liang, D.Juan, W.Li, H.Pan, L.Ding, H.Zeng. **A new Yb-doped oxyorthosilicate laser crystal: Yb : Gd₂SiO₅** .
– Solid State Comm.**137**, 451-455 (2006)
- D.E.McCumber, **Einstein relations connecting broadband emission and absorption spectra**. – Phys.Rev. **136** (4A), 954-957 (1964)
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- A. Shirakawa. Private communication, 2006.
- Editors, reviewers and other colleagues:
Emission cross-section of a material with different sites has no need to satisfy the McCumber relations!
- ?

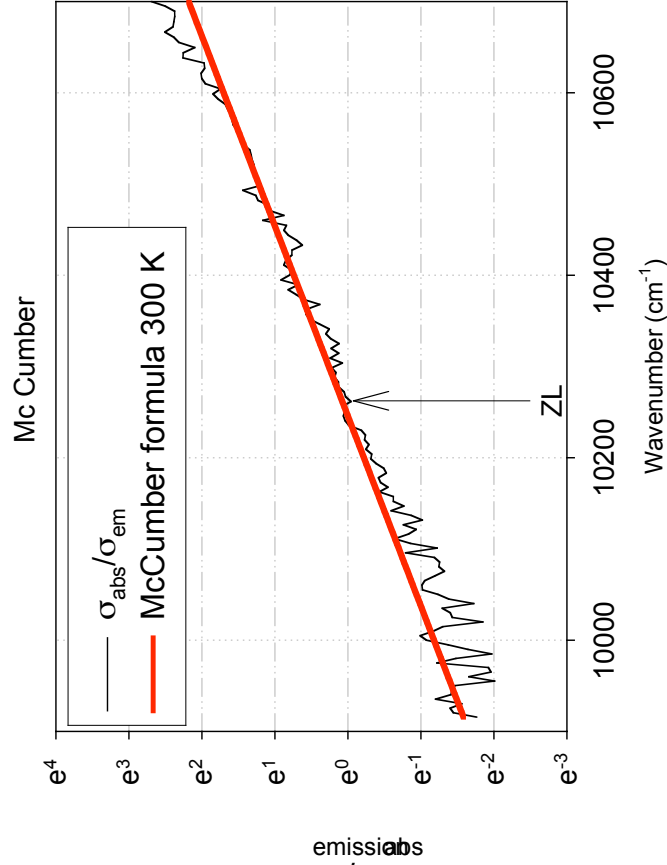
Absorption and Emission Cross-Section Yb:Lu₂O₃ceramics



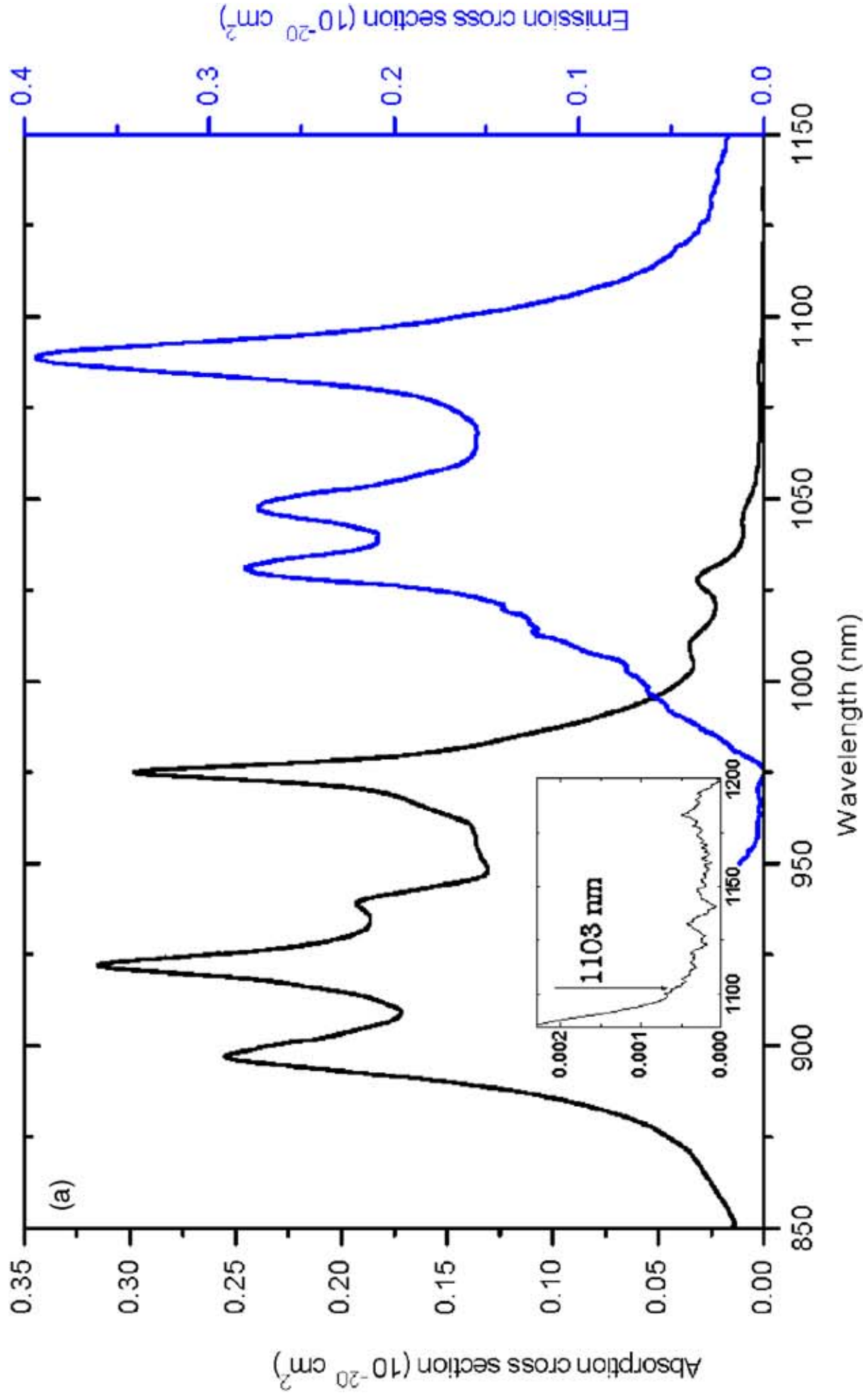
Example of a check of experimental data for the McCumber relation

Data for 3%Yb : Lu₂O₃
 by J.-F.Bisson et al., Optics Japan 2006,
 Tokyo

$$\frac{\sigma_a(\lambda)}{\sigma_e(\lambda)} = \exp \left[\frac{hc}{k_B T} \left(\frac{1}{\lambda} - \frac{1}{\lambda_{ZL}} \right) \right]$$



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Efficient diode-pumped Yb: Gd₂SiO₅ laser
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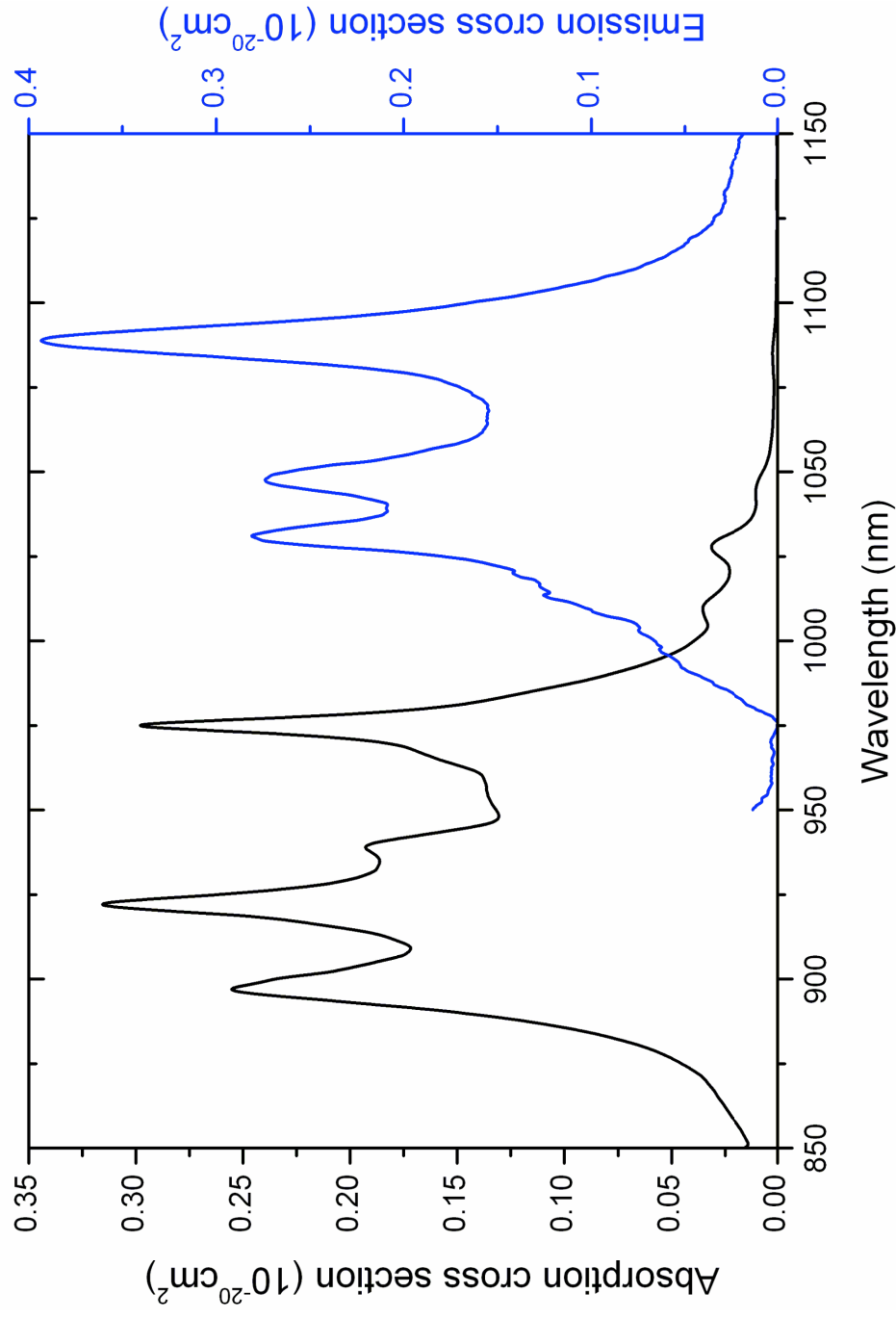


Fig. 1. Room-temperature absorption and emission spectra of the Yb:GSO laser crystal.

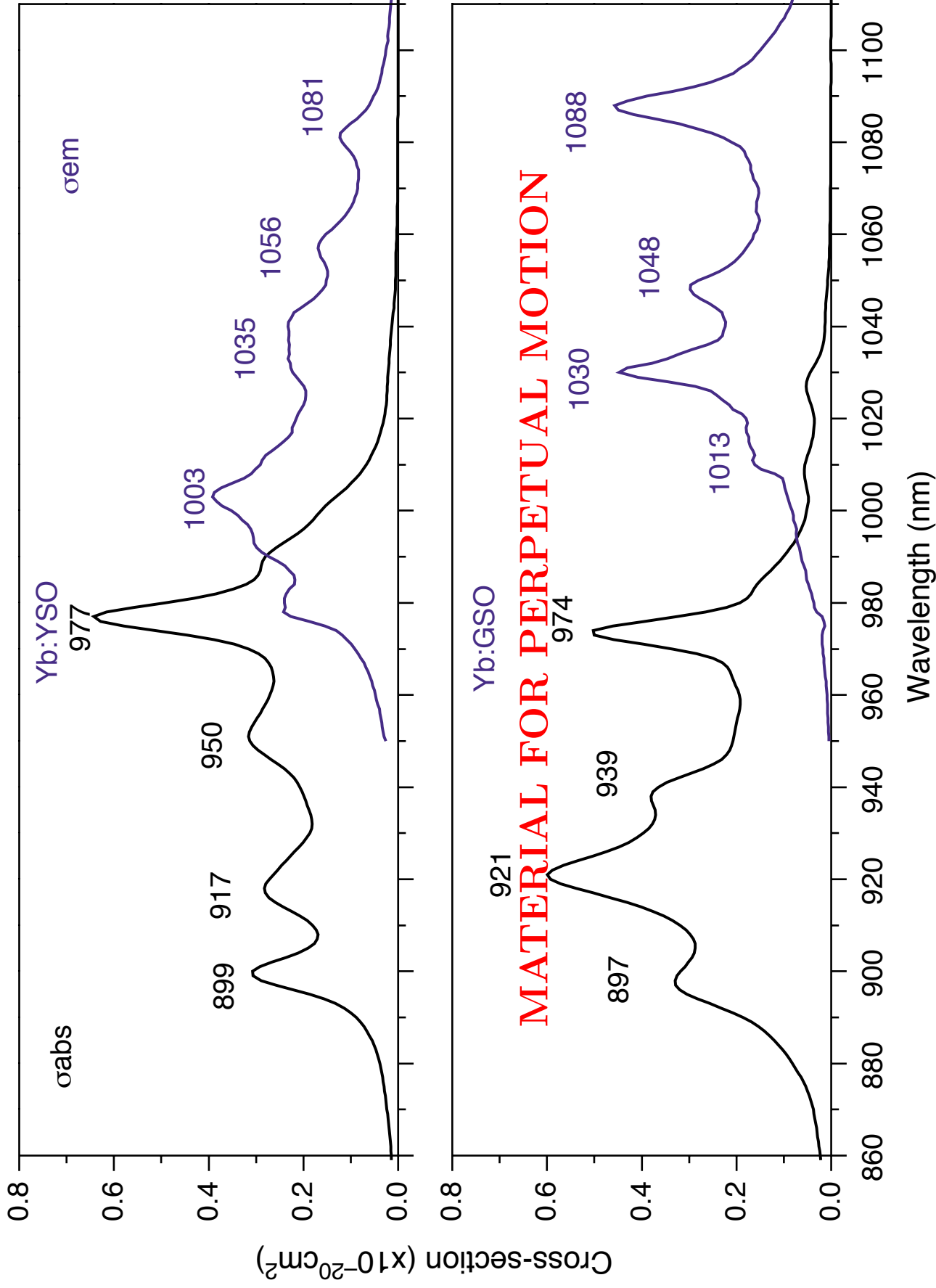
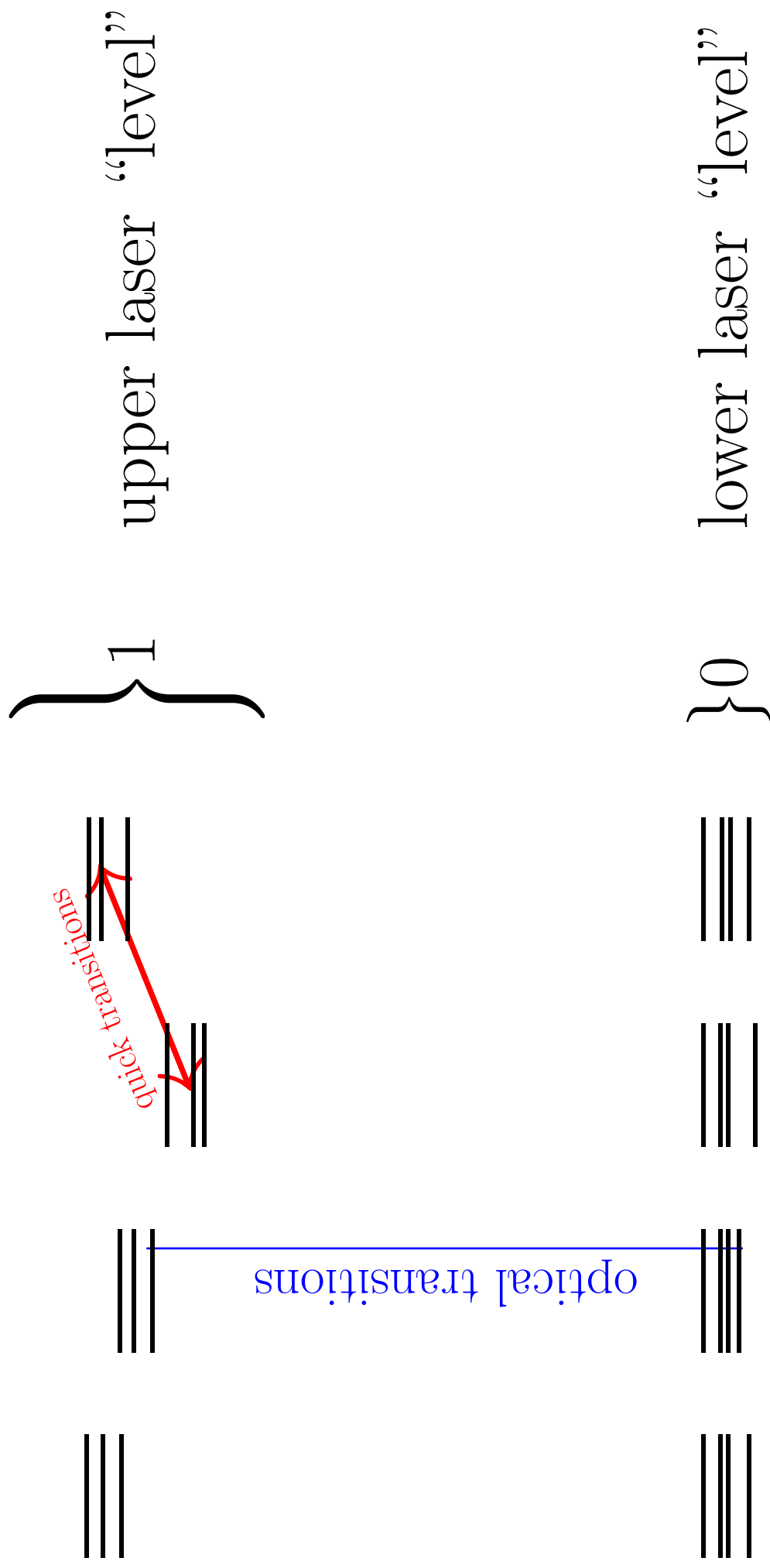


Fig. 3. The absorption cross section σ_{abs} and the emission cross section σ_{em} of 5 at.% Yb:GSO and 5 at.% Yb:YSO.

What is Emission cross-section ?

gain: $g(\omega) = n_1 \sigma_e(\omega) - n_0 \sigma_a(\omega)$



McCumber relation

$$\sigma_e(\omega) = \sigma_a(\omega) \exp\left(\frac{\hbar\omega_z}{K_B T} - \frac{\hbar\omega}{K_B T}\right)$$

Zero-line: $\sigma_a(\omega_z) = \sigma_e(\omega_z)$

The McCumber relation holds well for various Er, Yb and Nd doped materials

What about Yb : Gd₂SiO₅ ?

Deduction of the McCumber relation

Let N_m be number of photons in the m -th mode of an idealized box cavity;

$$\frac{dN_m}{dt} = (N_m + 1) n_1 v(\omega_m) \sigma_e - N_m n_0 v(\omega_m) \sigma_a ,$$

ω_m is frequency of the m th mode ; $v(\omega)$ is group velocity

conservation of number of excitations: $V n_1 + \sum_m N_m = \text{constant}$

In the steady-state: $(N_m + 1) n_1 v(\omega_m) \sigma_e = N_m n_0 v(\omega_m) \sigma_a$

Bose distribution:
$$N_m = \frac{1}{e^{\hbar\omega_m/(k_B T)} - 1}$$

then

$$\frac{\sigma_a(\omega)}{\sigma_e(\omega)} = \left(\frac{n_1}{n_0} \right)_T \exp\left(\frac{\hbar\omega}{k_B T} \right)$$

The McCumber relation is not a specific property of homogeneously-broadened media. It holds for any medium where the effective emission cross-section has sense.

Zero-line frequency ω_z

$$\left(\frac{n_1}{n_0}\right)_T = \exp\left(-\frac{\hbar\omega_z}{k_B T}\right)$$

Case of homogeneous medium:

$$\left(\frac{n_1}{n_0}\right)_T = \frac{\sum_{i=L}^{U-1} \exp\left(-\frac{\hbar\epsilon_i}{K_B T}\right)}{\sum_{i=0}^{L-1} \exp\left(-\frac{\hbar\epsilon_i}{K_B T}\right)} \approx \exp\left(-\frac{\hbar\epsilon_L}{K_B T}\right)$$

Case of a medium with different active sites; let q_s be concentration of s th site:

$$\left(\frac{n_1}{n_0}\right)_T = \frac{\sum_s q_s \frac{\sum_{i=L}^{U-1} \exp\left(-\frac{\hbar\epsilon_{s,i}}{K_B T}\right)}{\sum_{i=0}^{U-1} \exp\left(-\frac{\hbar\epsilon_{s,i}}{K_B T}\right)}}{\sum_s q_s \frac{\sum_{i=0}^{L-1} \exp\left(-\frac{\hbar\epsilon_{s,i}}{K_B T}\right)}{\sum_{i=0}^{U-1} \exp\left(-\frac{\hbar\epsilon_{s,i}}{K_B T}\right)}}$$

Numeration of sublevels:

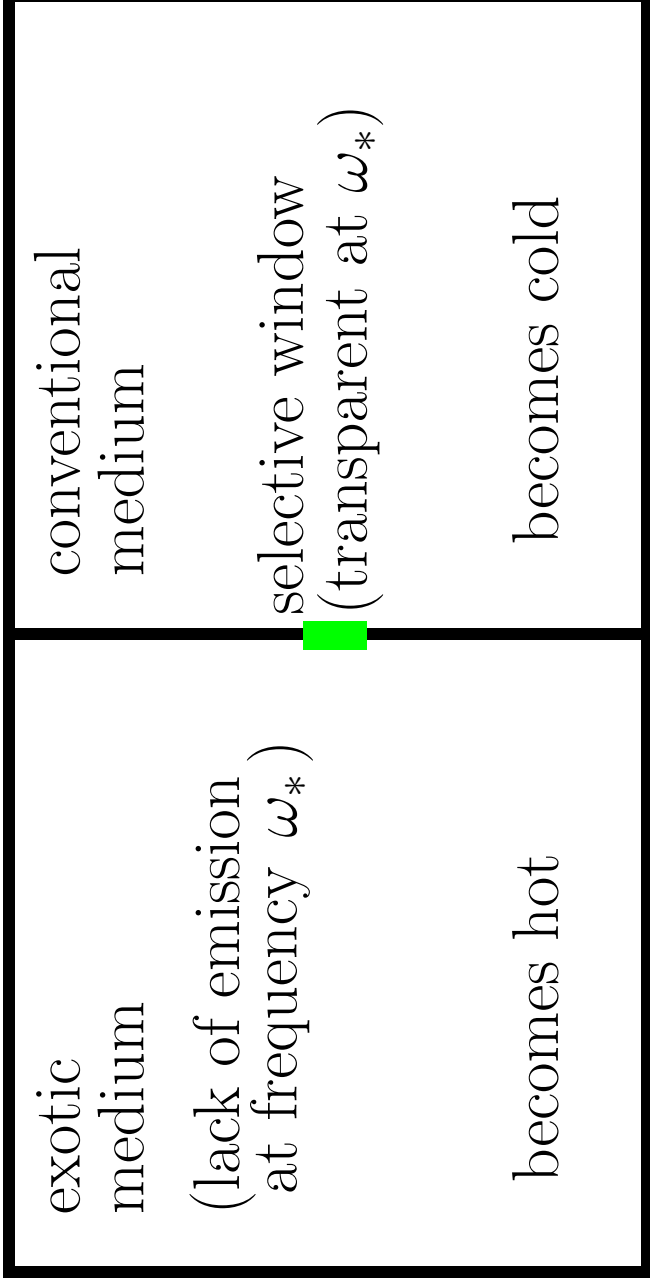
$$\begin{array}{l} \equiv \\ \equiv \\ \equiv \\ \equiv \end{array} \left. \begin{array}{l} U-1 \\ L \end{array} \right\} U-L \text{ upper sublevels}$$

$$\begin{array}{l} \equiv \\ \equiv \\ \equiv \\ \equiv \end{array} \left. \begin{array}{l} L-1 \\ 0 \end{array} \right\} L \text{ lower sublevels}$$

$$\omega_z = -\frac{k_B T}{\hbar} \ln \left(\frac{n_1}{n_0}\right)_T$$

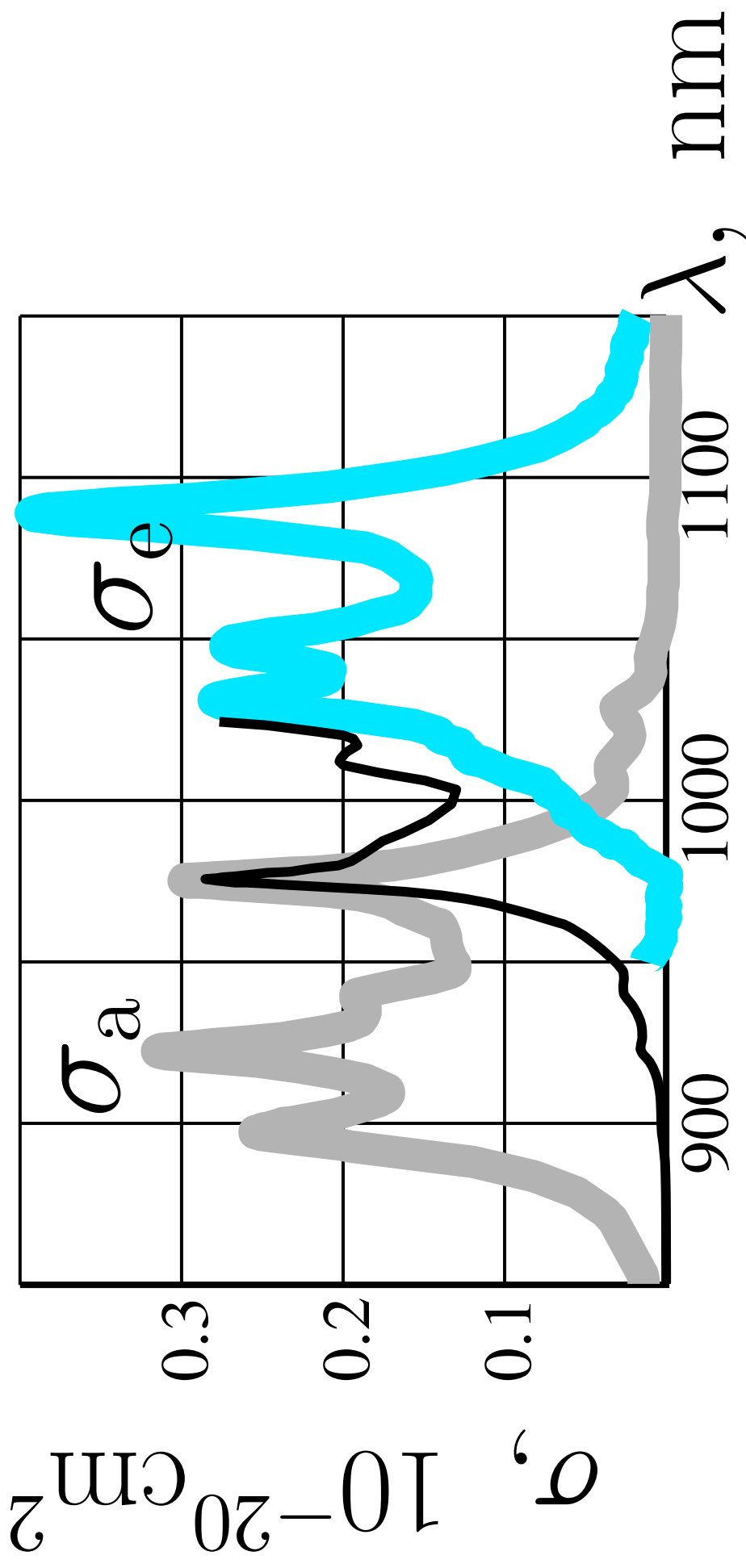
Gedanken experiment: perpetual motion

Assume that for some exotic medium, the emission and absorption cross-sections deviate from McCumber (at least in some narrow interval of frequencies)



If for some medium, in some approximation, both emission cross-section and absorption cross-section can be defined, **then** either these cross-section satisfy the McCumber or the medium allows the perpetual motion of Second kind.

Correction of σ_e for $\text{Yb}:\text{Gd}_2\text{SiO}_5$



Thick curves: σ by App.Ph.Let.**88**,221117(2006) and Opt.Express **14**,686-695(2006)

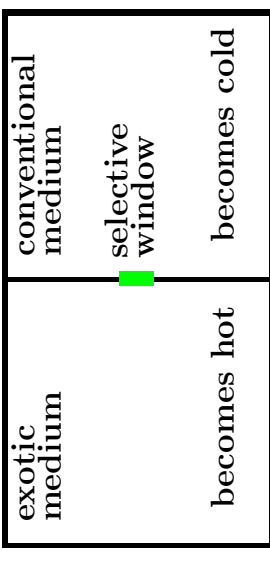
Thin black curve: σ_e , reconstructed with McCumber from σ_a .

Values used for reconstruction: $T = (271 + 14)$ Kelvin, $\omega_z = \frac{2\pi c}{975 \text{ nm}}$

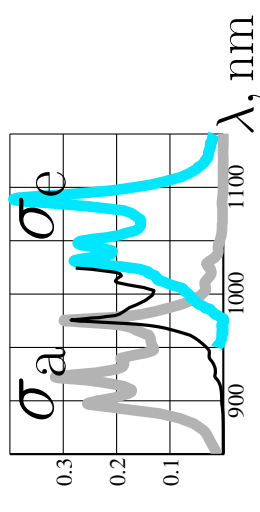
Conclusions

$$\frac{\sigma_a(\omega)}{\sigma_e(\omega)} = \left(\frac{n_1}{n_0}\right)_T \exp\left(\frac{\hbar\omega}{k_B T}\right)$$

McCumber relation applies not only to homogeneous laser media, but to any medium with fast transitions within groups of sub-levels. If the concept of emission cross-section has sense for some medium, then, for this medium, the McCumber holds. Any medium with effective cross-sections, violating the McCumber relation, allows the perpetual motion of Second kind.



Spectra of Yb : Gd₂SiO₅ by Appl.Phys.Lett and Optics Express cannot be interpreted as emission cross-section. Such interpretation contradicts the Second Law. If σ_a of Yb : Gd₂SiO₅ was measured correctly, then σ_e can be reconstructed with McCumber.



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W.Li, H.Pan, L.Ding, H.Zeng, G.Zhao, C.Yan, L.Su, J.Xu. Diode-pumped continuous-wave and passively mode-locked Yb : Gd₂SiO₅ laser. Optics Express 14, 686-695 (2006)