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Reflection of Waves from a Ridged Surface and the Zeno Effect

Dmitrii KOUZNETSOV* and Hilmar OBERST

Institute for Laser Science, University of Electro-Communications, 1-5-1 Chofugaoka, Chofu, Tokyo 182-8585, Japan

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Reflection of waves from a ridged surface is interpreted as the spatial Zeno effect. This interpretation leads to a simple estimate for the coefficient of reflection. This estimate shows good agreement with experimental data of scattering of cold atoms from ridged surfaces.

Key words: Zeno effect, quantum reflection, specular reflection, cold atoms, atomic mirror, atom optics, screen problem

1. Introduction

Waves are reflected at any abrupt change in the index of refraction, regardless of the sign of change. In the case of light waves, such step in the index of refraction arises naturally at any interface of two media with different optical properties. In the case of atomic waves, the reflection occurs when the interaction potential changes significantly within one de Broglie wavelength. The phenomenon has been called "quantum reflection" because it relies on the wave-like properties of atoms.

The classical specular reflection of atoms from condensed matter is usually impossible because of the attractive character of the van der Waals potential. When an atom approaches so close that the potential becomes repulsive, it is already well accelerated and feels the discrete structure of the surface. This leads to diffuse scattering. In contrast, the quantum reflection occurs at a distances large in comparisson to the atomic size, and leads to specular reflection.

The quantum reflection has been predicted and observed with ultra-cold atoms, and then intensively studied as a way to realize stable, accurate and dispersionless atom reflectors.[1–12] However, the quantum reflection quickly decreases with increasing normal wavenumber, and the application is usually limited to ultra-cold atoms and/or small grazing angles.

The reflectivity of atoms is higher when the van der Waals interaction potential is weaker. This indicates that the van der Waals interaction prevents efficient reflection, because it accelerates the atoms toward the surface. A material with lower density would be a better reflector. This reasoning motivated Shimizu and Fujita to use nano-fabricated silicon surface structures consisting of parallel narrow ridges to reflect cold atoms.[14,15] A schematic cross-sectional view of such ridges is shown in Fig. 1(a). In this experiment, the laser-cooled metastable neon atoms were incident at a small grazing angle θ on this surface structure and were reflected much better than on a smooth surface.

In spite of this progress, the process that leads to such increase of the reflectivity on the grating structures was not yet understood. First, the enhancement in reflectivity was estimated by assuming a reduction of the density, and hence the van der Waals constant, by a factor ℓ/L. This assumption overestimates the reflectivity and does not allow prediction of properties of the ridged mirrors.[14)]

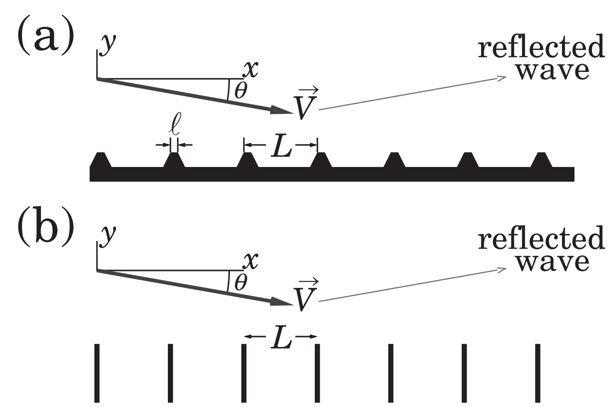

Fig. 1. (a) Particle with velocity V incident at grazing angle θ onto a surface with a grating structure. (b) The idealization of the structure as a periodical set of absorbers.

Here, we present a simple model which is in a good agreement with experimental results performed on a ridged surface. In this model, the reflectivity is determined by the dimensionless parameter

$$p^2 = 2E_{\rm nor}/(\hbar f) \qquad (1)$$

where $E_{\rm nor} = mv^2/2$ is kinetic energy of an atom of mass m, corresponding to the normal component v of its velocity; V is tangential velocity, and $f = V/L$ is frequency at which the atom passes by the ridges. Parameter p has sense of normalized momentum.

Our model interprets the ridges as idealized absorbers as shown in Fig. 1(b). Let x and y be Cartesian coordinates. Atoms are incident onto the ridged surface under a small grazing angle θ. The grazing angle θ in experiments[14–16] with ridged atomic mirrors was in the order of 10 mrad, and the height of the ridges were high enough that the incoming wave saw mainly the side-walls of the ridges. Assuming small values of the grazing angle θ, we make no difference between the incident velocity and its tangential component V. The narrow ridges are placed with period L. These ridges either absorb the atoms or modify their quantum state.

*E-mail address: dima@ils.uec.ac.jp

Neither the width ℓ of the ridges [Fig. 1(a)], nor the potential of interaction of atoms with the surface are used in our model. The transversal velocity of the incoming atoms is high so that they are expected to collide with the ridge walls. The metastable atoms are quenched during such a collision. In the following section we interpret this quenching as "measurement"; then we show that such interpretation leads to a simple and correct estimate.

2. Reflection from Ridges as Spatial Zeno Effect

In general, the Zeno effect can be defined as *a class of phenomena when a transition is suppressed by some interaction which allows the interpretation of the final state in terms of "a transition has not yet occured" or "a transition already occurred"*. In quantum mechanics, such an interaction is called "measurement" because its result can be interpreted in terms of classical mechanics. Frequent measurement prohibits the transition. Various versions of the Zeno effect[17–21] fall into our definition above. We need to justify the term "measurement" in the case of interaction of atoms with ridges. In this section, we apply the concept of Zeno effect to the transition of the atom from the upper half-space ($y > 0$) to the lower half-space ($y < 0$).

In principle it is possible to detect the collision of an excited atom with the wall of a ridge. The high internal energy of metastable noble gas atoms typically leads to the emission of an electron during the collision. Also, the atom could emit a photon, or just exchange momentum and energy with a phonon at the surface—any kind of interaction which removes the particle (atom) from the beam. We therefore interpret the ridges as a measurement device that tells us whether the atom has already collided with the surface or not. The good detector could even say which particular ridge has just been hit by the atom. We have a grating with many ridges along the surface, so the position of the atom is periodically measured. The rate of measurement is given by the velocity V of the atoms along the surface and the distance L between the ridges. This is the spatial Zeno effect; the frequently repeated measurement of the atom in the lower half-space prohibits the transition. Such prohibition means reflection.

Although the Zeno effect was formulated far before quantum mechanics, the reflection caused by the measurement is a relatively new phenomenon. Last century, such a reflection looked counter-intuitive and caused worries that the statistical interpretation of the quantum mechanical wave packet contained a gap.[22] In our paper, we use the concept of spatial Zeno effect for quick interpretation of recent experimental data[14–16] of the reflection of excited He and Ne atoms from ridged surfaces. These data appear as the experimental evidence of the spatial Zeno effect.

The parameter p^2 by eq. (1) can be interpreted as doubled ratio of frequency of the system $\omega = E_{\rm nor}/\hbar$ to the frequency f of the measurement. In our case, the frequency ω is the only frequency which characterizes the movement of the atom toward the surface. If the frequency f of the measurements is large compared to ω, we expect the suppression of the absorption due to the Zeno effect. Therefore, the mormalized momentum p is the key parameter which determines the reflectivity of the ridged mirror; at $p \ll 1$, we expect strong reflection, and at $p \gg 1$ we expect strong absorption. From eq. (1) we see that the Zeno-reflection of macroscopical objects requires too high frequency of measurements. In the case of cold atoms, however, such reflection is possible and we estimate its efficiency below.

The efficiency of the reflection is the same, whether the ridges are connected to some macroscopic measurement device or not. It is sufficient that the state of an atom entering the half-space $y < 0$ gets entangled with some sub-system. Such a sub-system can be the emitted electron or photon, as well as phonons in the ridges. Then, the actual measurement *could* be performed without any additional distortion of the state of the atom. In this sense, the frequent absorption causes the reflection, and the reflectivity is determined by the frequency of the detection.

Once we accept the interpretation of the reflection from a ridged surface as the Zeno effect, we can simplify the derivation by assuming that the measurement at a rate f with unit detection efficiency is equivalent to a continuous measurement with absorption rate f. At grazing incidence we therefore expect the grating structure to behave like an equivalent absorbing medium with an absorption rate $f = V/L$, where V is tangential component of velocity. This strong assumption allows us to obtain an easy analytical estimate of the reflection coefficient R as function of dimension-less momentum p.

3. Reflectivity by Continuous Detection

Following the ideas of the previous sections we interpret the ridged surface as a set of detectors, then as a distributed detector with the same rate of detection, and then as an equivalent absorbing medium with absorption rate f. Such assumption allows us to estimate the reflection coefficient.

Let Ψ be the part of the scattering wave function which represents the atom that has not yet interacted with the detector. Assume that the half-space $y < 0$ is filled with a continuous detector which detects (absorbs) the atom with rate $f = V/L$. Then we can consider the single-dimensional Schrödinger equation for wave function Ψ of the movement in y-direction:

$$i\frac{\partial\Psi}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2\Psi}{\partial y^2} - i\frac{f}{2}\vartheta(-y)\Psi \tag{2}$$

where ϑ is the unit-step function. The last term phenomenologically takes into account the entanglement of the matter wave with other degrees of freedom. This entanglement occurs with the rate f in the half-space ($y < 0$).

Assuming the monochromatic beam, we write

$$\Psi = \Psi(y,t) = \psi(y)\exp(-i\omega t) \tag{3}$$

where ψ is spatial part of the wave function. It satisfies the equation

$$\left(\frac{d^2}{dy^2} + k^2 + i\gamma^2\vartheta(-y)\right)\psi = 0, \tag{4}$$

where $k = mv/\hbar$ is the transversal component of wavenumber, m is the mass of the atom, v is normal component of velocity, and $\gamma^2 = fm/\hbar$.

We construct the solution of (4) in the form:

$$\psi = \begin{cases} e^{-iky} - re^{iky}, & y \geq 0 \\ (1-r)e^{(-i\alpha-\beta)y}, & y \leq 0 \end{cases} \tag{5}$$

where r is the complex reflection amplitude, and α and β are positive parameters. Outside the absorber, the wave consists of the incoming and reflected parts. Inside the absorber, the wave function decays exponentially, as if it were be an optical wave inside an absorbing medium.

Substitution of (5) into (4) gives the equations for α and β:

$$\alpha^2 - \beta^2 = k^2 \tag{6}$$

$$2\alpha\beta = \gamma^2 \tag{7}$$

The appropriate solution is

$$\alpha = \sqrt{\frac{1}{2}\left(\sqrt{k^4+\gamma^4}+k^2\right)}, \tag{8}$$

$$\beta = \sqrt{\frac{1}{2}\left(\sqrt{k^4+\gamma^4}-k^2\right)}. \tag{9}$$

An equation for the complex reflection amplitude r is readily obtained using the continuity of the derivative of $\psi(y)$ at $y = 0$:

$$r = \frac{i\alpha+\beta-ik}{i\alpha+\beta+ik}. \tag{10}$$

Then, after some algebra, we express the reflection coefficient rr^* in the following form:

$$rr^* = R(p) = \frac{\sqrt{\sqrt{1/p^4+1}+1}-\sqrt{2}}{\sqrt{\sqrt{1/p^4+1}+1}+\sqrt{2}} \tag{11}$$

where

$$p = \sqrt{\frac{m}{\hbar f}}V\theta; \tag{12}$$

has the meaning of a dimension-less transversal momentum. In terms of wavenumber $K = mV/\hbar$, this parameter can be expressed even more simply:

$$p = \sqrt{\frac{2E_{\rm nor}}{\hbar f}} = \sqrt{\frac{mL}{\hbar V}}v = \sqrt{KL}\theta \tag{13}$$

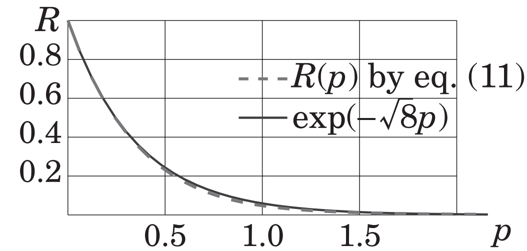

Fig. 2. Function $R(p)$ by eq. (11) (dashed line) and the fit $\exp(-\sqrt{8}p)$ (solid line).

The function $R(p)$ by (11) can be approximated by the exponential fit $\exp(-\sqrt{8}p)$; the absolute difference between these two functions does not exceed 0.02%. Both curves are shown in Fig. 2. The reflectivity estimated by eq. (11) is determined by the mormalized momentum p, which demonstrates the scaling law for the reflectivity of waves on a ridged surface.

4. Comparison with Experimental Data

In this section, we compare our estimate with various experimental data for the reflection of cold atoms from ridged silicon surfaces. Figure 3 collects the data from Fig. 3 of ref. 14. and Fig. 4(b) of ref. 16 We plot the reflectivity measured on various samples versus the normalized momentum p by eq. (13). The dashed curve represents the estimate (11), and the solid line is the fit $\exp(-\sqrt{8}p)$.

Our estimate explains the almost exponential decay of the reflection coefficient as function of the grazing angle observed in experiments. In addition, it predicts the correct slope of this decay. Equation (11) shows good agreement with the data for Ne* and He* atoms in a wide range of values of parameters. The mass m changes from 3.4×10^{-26} to 0.7×10^{-26} kg, the distance L between ridges varies from 5 to 100 µm, shape changes from trapesoidal[14] to almost rectangular,[16] the width ℓ of the ridges changes from 40 nm to 11 µm, the atom velocity V varies from 3 to 126 m/s, and the transversal velocity v varies from 4 mm/s to 150 cm/s; but our estimate holds. Therefore we interpret the measurements mentioned as the experimental evidence of the spatial Zeno effect.

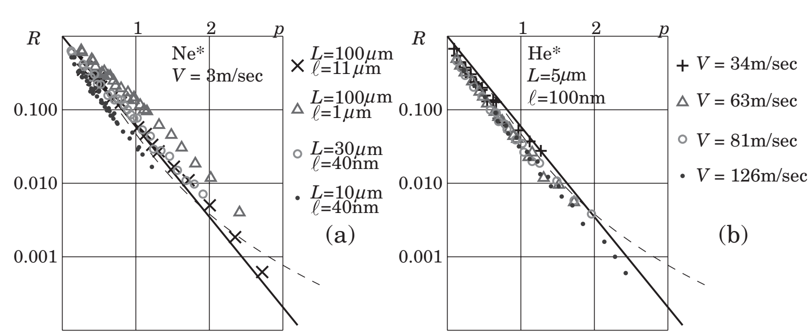

Fig. 3. (a): Reflectivity of Ne* with velocity $V = 3$ m/sec from various ridged Si samples.[14] (b): Similar data from ref. 16 for He* atoms scattered from the sample with $L = 5$ µm, $\ell = 100$ nm.

5. Limits of the Model

The assumption of a continuous absorbing medium is a rough model of the ridged surface. First, it ignores the periodical character of absorbers, so it cannot be used for analysis of higher orders of scattering. Second, it cannot predict the effect of the finite width ℓ of the ridges. These two effects may partially compensate each other, at least in the case of our samples.

As we reduce the width ℓ of the ridges [Fig. 1(a)], the reflectivity increases (compare the data for $L = 100\,\mu\text{m}$, $\ell = 11\,\mu\text{m}$ to the data for $L = 100\,\mu\text{m}$, $\ell = 1\,\mu\text{m}$) and exceeds the dashed curve in Fig. 3(a). Hence, the expression (11) slightly underestimates the reflectivity of a set of idealized absorbers. This limitation can be revealed using different kinds of waves (neutrons, photons) which do not feel the van der Waals attraction. It is interesting to see how the estimate (11) works for other kinds of waves.

For the excited He and Ne atoms, the description of reflection as spatial Zeno effect catches the most important properties of the phenomenon and provides a quick estimate of the reflectivity. However, the speculations about measurements do not bring any effect that would not follow from the wave equations directly. The same equations and estimate (11) can also be obtained from the assumption that L is the effective absorption length of atoms in the half-space occupied with ridges. More detailed consideration[23] takes into account also the discrete character of absorbers and the van der Waals interaction, providing even more precise estimates of the reflectivity.

6. Conclusion

We have estimated the reflectivity of atoms on a surface structure with narrow ridges. In order to get such an estimate, we interpret the ridges as idealized detectors. This allows us to describe the reflection process in terms of the Zeno effect. Then we assume that the discrete detectors are equivalent to a distributed detector. For the scattering wave function, such a detector appears as distributed absorber placed in the lower half-space (Fig. 1). This leads to an estimate of the reflection coefficient in terms of an elementary function (11).

In this analysis, we have ignored the finite width of the ridges and the van der Waals interaction between the atoms and the surface. Nevertheless, our estimate explains the almost exponential decay of the reflectivity with increase of the grazing angle and shows good agreement with experimental data over a wide range of parameters. This agreement confirms our interpretation of the reflection of atoms from a ridged mirror as the spatial Zeno effect.

We expect this result to be useful for the design of ridged mirrors for various applications including optics of atomic beams.

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