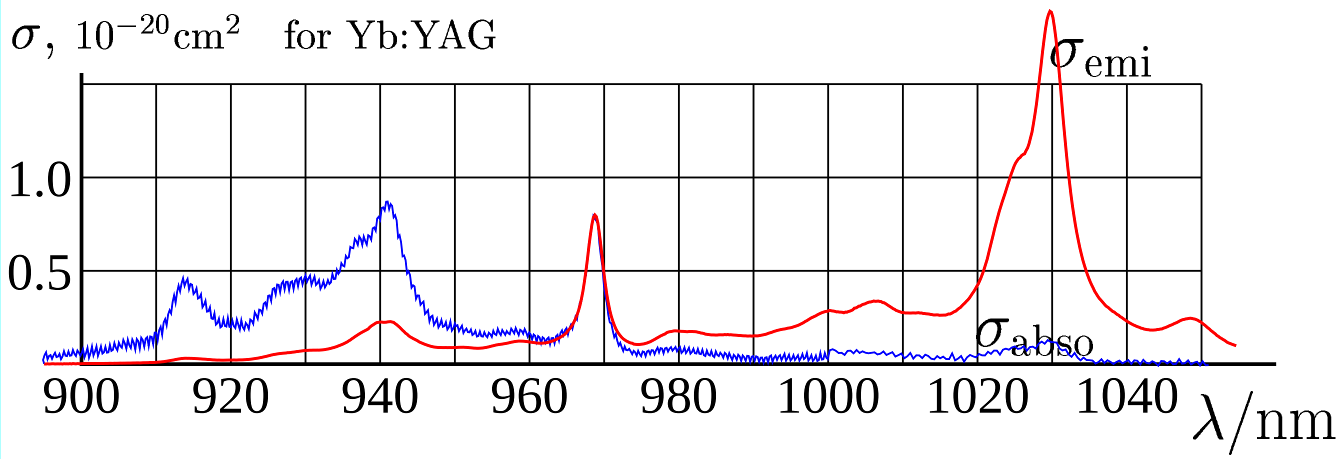
ABSORPTION AND EMISSION SPECTRA. McCUMBER RELATION

Motivation:

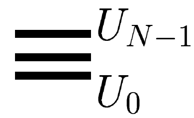
In order to predict properties of laser devices,
we need to know the absorption and emission cross-sections.

Should they be measured independently for each temperature?



Data by Bisson and Takaichi for $T = 300$Kelvin

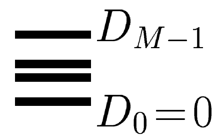
McCumber Theory



$\beta = 1/(k_{\mathrm{B}}T)$; thermal distribution within bands:

$$f_n = F e^{-\beta U_n} \text{ , } n = 0..N-1 \text{ ; } F = 1/\sum_{n=0}^{N-1} e^{-\beta U_n}$$

$$g_m = G e^{-\beta D_m} \text{ , } m = 0..N-1 \text{ ; } G = 1/\sum_{m=0}^{M-1} e^{-\beta D_m}$$



Fundamental rule for particular cross-sections:

$\sigma_{\mathrm{abso},m,n}(\omega) = \sigma_{\mathrm{emi},m,n}(\omega) = \sigma_{\mathrm{emi},m,n}(\omega)$

Then, the effective cross-sections

$$\sigma_{\mathrm{abso}}(\omega) = \sum_{m=0}^{M-1} g_m \sum_{n=0}^{N-1} \sigma_{m,n}(\omega) = G \sum_{m,n} e^{-\beta U_n} \sigma_{m,n}(\omega)$$

$$\sigma_{\mathrm{emi}}(\omega) = \sum_{n=0}^{N-1} f_n \sum_{m=0}^{M-1} \sigma_{m,n}(\omega) = F \sum_{m,n} e^{-\beta D_m} \sigma_{m,n}(\omega)$$

Assume that spectral width $\delta\omega$ of each transition is small, only terms with $\hbar\omega \approx U_n - D_m$ contribute. Replace U_n to $D_m + \hbar\omega$

$$\sigma_{\mathrm{abso}}(\omega) \approx G \sum_{m,n} e^{-\beta(D_m+\hbar\omega)} \sigma_{m,n}(\omega) = G e^{-\beta\hbar\omega} \sum_{m,n} e^{-\beta D_m} \sigma_{m,n}(\omega)$$

General relation: $\sigma_{\mathrm{abso}}(\omega) \approx \dfrac{G}{F} e^{-\beta\hbar\omega} \sigma_{\mathrm{emi}}(\omega)$

$$\frac{G}{F} e^{-\beta\hbar\omega} = \frac{\sum_n e^{-\beta\hbar U_m}}{\sum_m e^{-\beta\hbar D_n}} e^{-\beta\hbar\omega} = \frac{\sum_n e^{-\beta\hbar(U_n-U_0)}}{\sum_m e^{-\beta\hbar D_m}} e^{\beta(\hbar\omega-U_0)}$$

$$\frac{G}{F} e^{-\beta\hbar\omega} = \frac{1 + \sum_{n>0} e^{-\beta\hbar(U_n-U_0)}}{1 + \sum_{m>0} e^{-\beta\hbar D_m}} \; e^{\beta(\hbar\omega-U_0)}$$

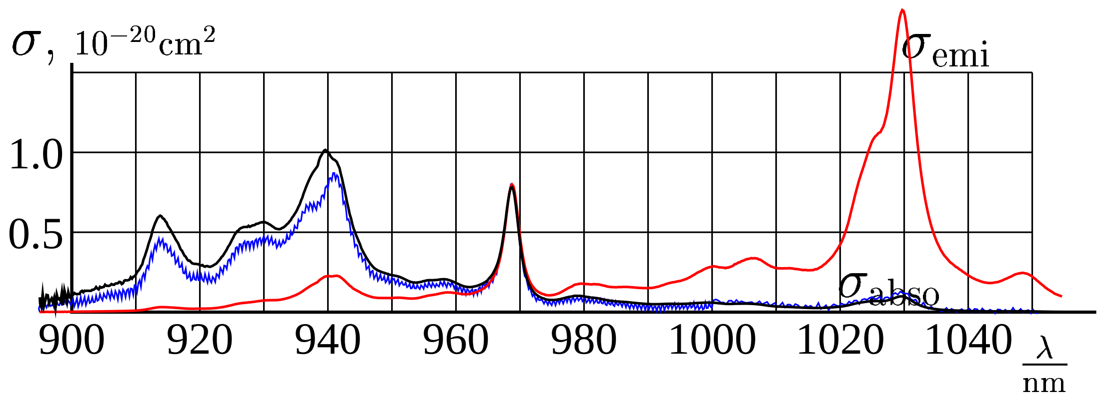
$$\sigma_{\mathrm{abso}}(\omega) \approx \exp\left[\frac{\hbar(\omega-\omega_0)}{k_{\mathrm{B}}T}\right] \sigma_{\mathrm{emi}}(\omega)$$

frequency $\omega_0 \approx U_0/\hbar$ can be called "zero-line";

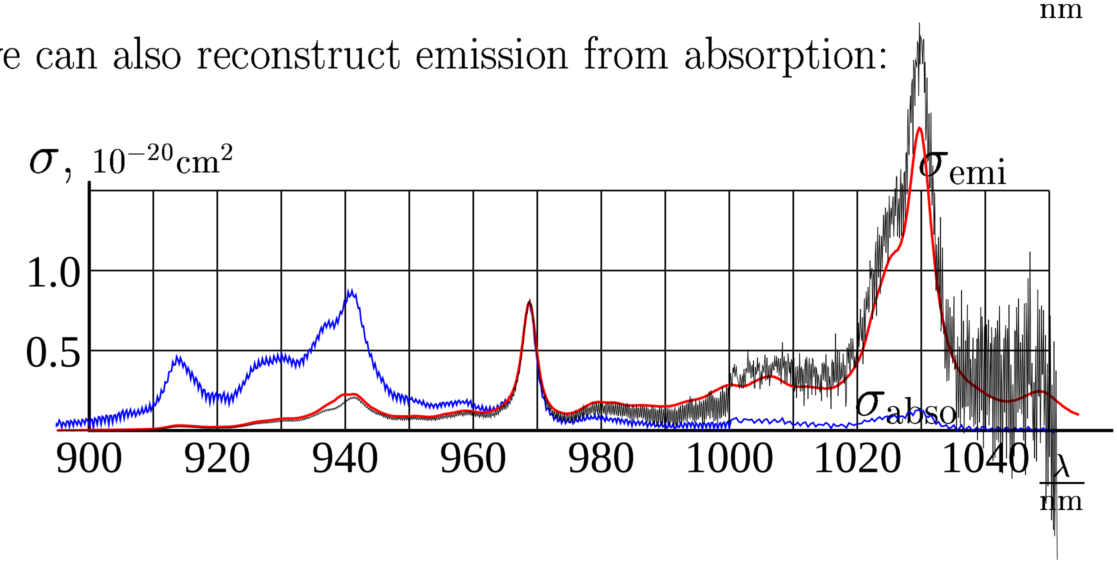
At zero line, $\sigma_{\mathrm{abso}} = \sigma_{\mathrm{emi}}$

Historically, graphics are plotted versus $\lambda = \dfrac{2\pi\hbar c}{\omega}$

$$\sigma_{\text{abso}}(\omega) \approx \exp\left[\frac{\hbar(\omega - \omega_0)}{k_{\text{B}}T}\right] \sigma_{\text{emi}}(\omega) \quad ; \quad \omega = \frac{2\pi\hbar c}{\lambda}$$



we can also reconstruct emission from absorption:



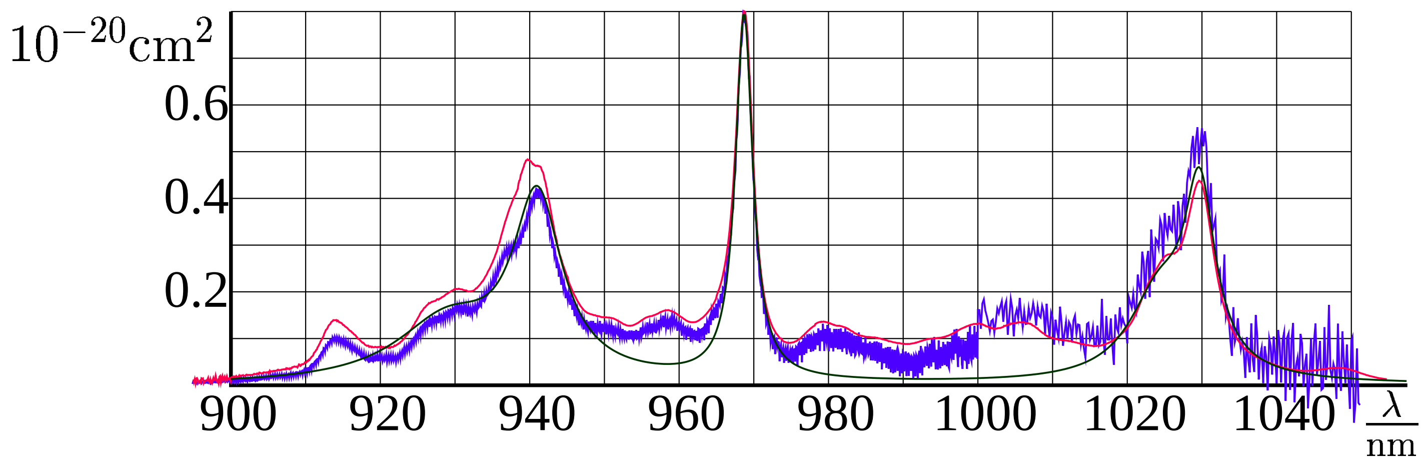
In order to see cross-sections in the same scale, plot

$$\sigma_{\text{abso}}(\omega)\exp\left[\frac{\hbar(\omega-\omega_0)}{2k_{\text{B}}T}\right] \quad \text{あお} \quad , \qquad T = 300\text{Kelvin}$$

$$\sigma_{\text{emi}}(\omega)\exp\left[-\frac{\hbar(\omega-\omega_0)}{2k_{\text{B}}T}\right] \quad \text{あか} \quad ; \qquad \omega_0 = c10322\text{cm}^{-1} = 3\times10^{14}\text{Hz}$$

$$\sigma_{\text{fit}} = 10^{-20}\text{cm}^2\ \text{fit}\left(\frac{\lambda}{\text{nm}}\right) \quad \text{くろ} \quad , \text{ made of } b(x_0,w,x) = \frac{1}{1+\left(\frac{1/x-1/x_0}{w/x_0^2}\right)^2}$$

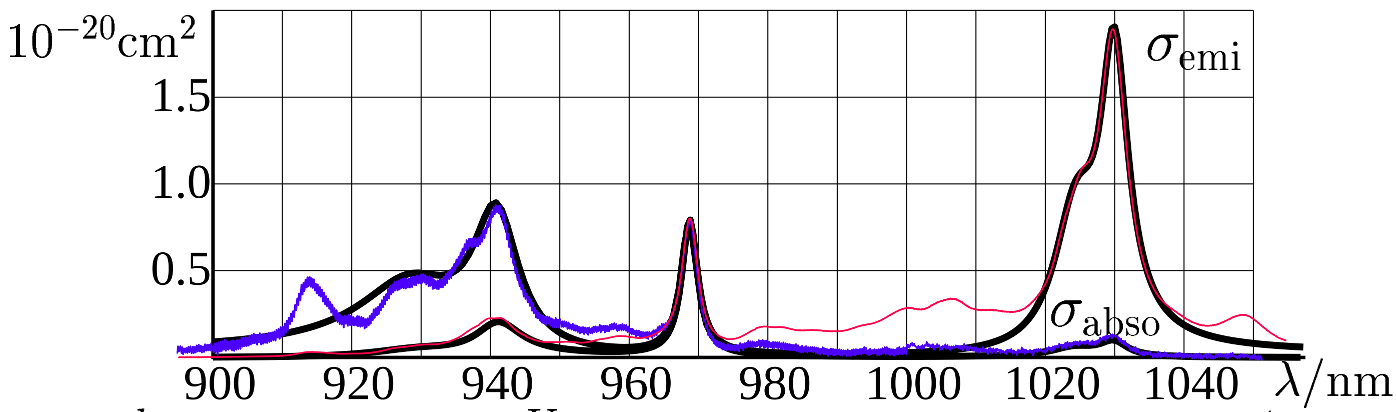
$$\text{fit}(x) = .13b(929.,8.5,x) + .38b(941.,4.,x) +$$
$$+.78b(968.7,1.4,x) + .18b(1024.3,5.,x) + .38b(1029.7,2.5,x)$$



$\sigma_{\rm abso}(\omega)$ あお , $\sigma_{\rm emi}(\omega)$ あか , data by Bisson, Takaichi

$\sigma_{\rm abso,fit} = 10^{-20}{\rm cm}^2\ {\rm fit}\left(\frac{\lambda}{\rm nm}\right) \exp[\lambda_T/\lambda - \lambda_T u_0]$ くろ

$\sigma_{\rm emi,fit} = 10^{-20}{\rm cm}^2\ {\rm fit}\left(\frac{\lambda}{\rm nm}\right) \exp[-(\lambda_T/\lambda - \lambda_T u_0)]$ くろ



$$\lambda_T = \frac{\pi\hbar c}{k_{\rm B}T} = 24\mu{\rm m}\,,\quad u_0 = \frac{U_0}{2\pi\hbar c} = 10322/{\rm cm}\,,\ b(x_0,w,x) = \frac{1}{1+\left(\frac{1/x-1/x_0}{w/x_0^2}\right)^2}$$

$${\rm fit}(x) = .13b(929.,8.5,x) + .38b(941.,4.,x) + .78b(968.7,1.4,x) + \\ + .205b(1024.3,3.9,x) + .38b(1029.9,2.4,x)$$

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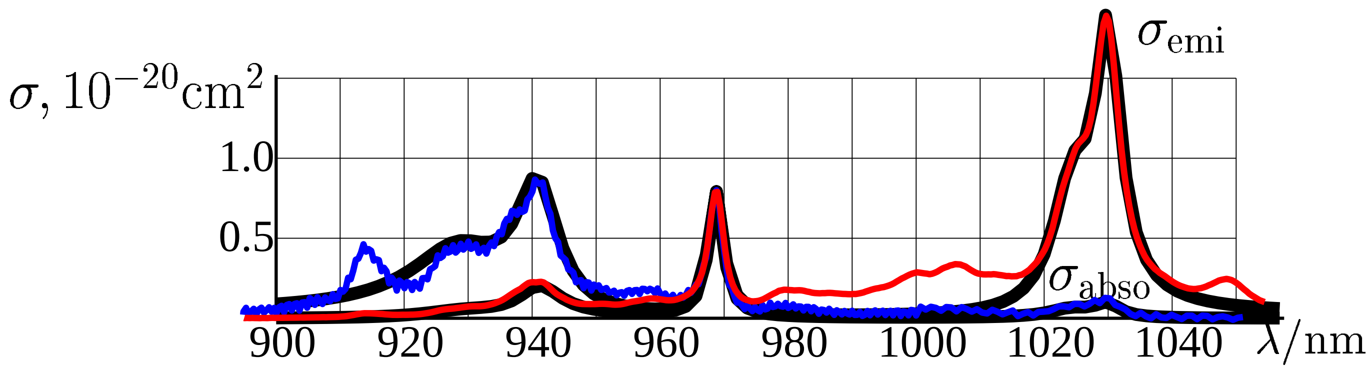
The McCumber relation provides good check of self-consistency of spectra of emission and absorption.

Both emission and absorption spectra are approximated with single fit. Does it capture some dependence on temperature?

More detailed analysis should take into account values D_m, U_n of energies of sub-levels. Low temperature spectra?



For detail analysis, we need energies of sub-levels. For Yb^{3+}:YAG

ref.	D_0	$\frac{D_1 \ {\rm cm}}{2\pi\hbar c}$	$\frac{D_2 \ {\rm cm}}{2\pi\hbar c}$	$\frac{D_3 \ {\rm cm}}{2\pi\hbar c}$	$\frac{U_0 \ {\rm cm}}{2\pi\hbar c}$	$\frac{U_1 \ {\rm cm}}{2\pi\hbar c}$	$\frac{U_2 \ {\rm cm}}{2\pi\hbar c}$
Basiev	0	565	612	875	10327	10624	10930
Brusselbach	0	586	613	776	10314	10627	10930
Kaminski	0	586	613	776	10327	10624	10679
Hanna	0	586	613	776	10327	10624	10940

T.T.Basiev, Yu.K.Voron'ko, T.G.Mamedov, I.A.Shcherbakov. Migration of energy between Yb^{3+} ions in garnet crystals. – Sov.J. of QE, v.15, No.10, p.1182-1188 (1976)

H.W.Bruesselbach, D.S.Sumida, R.A.Reeder, R.W.Byren. Low-heat high power scaling using InGaAs-diode pumped Yb:YAG lasers. – IEEE J.of QE, v.3, No.1, p.105-115 (1997)