Interaction of a spatially modulated wave of complex structure with a plane wave in a quantum amplifier

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A theoretical investigation is made of conversion (in an amplifier) of the image of a signal consisting of a wave with a pseudorandom transverse structure and a plane wave. Equations are obtained for the description of the mutual influence of these waves under saturation conditions. The results of calculations demonstrate a considerable change in the ratio of the intensities of these waves as a result of amplification.

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1. INTRODUCTION

Amplification of images in active optical systems[1] gives rise to distortions of spatial signals in an amplifying medium. In principle, such distortions are always possible and they may originate from the saturation effect in the amplifying medium. Particularly strong distortions are experienced by a signal when strong amplification is concentrated in a short distance.[2] When the amplifier length is increased, the relative proportion of distortions should decrease, but in this case it is n longer possible to carry out an analysis for an arbi form of the signal.

A method is developed in Ref. 3 for investigating propagation of a signal with a complex spatial struc in a nonlinearly amplifying medium. The differenti gain is found as a function of the intensity of a wave a pseudorandom spatial structure but without a con component. As expected, the value of the gain in th

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nonlinear regime is found to be less than the corresponding value for a plane wave. However, if the signal includes both an alternating pseudorandom component and a constant one, one may expect these two components to grow at different rates and thus distort the original signal. The presence of alternating and constant components in a signal is frequently encountered in real cases. In fact, when light is reflected from an object or when it is transmitted by a transparency, a likely result is a fairly strong constant component. Moreover, we can expect the appearance of various strays in an amplifier and these strays may include a fairly strong plane wave traveling along the axis of the system. Changes in the ratio of the alternating and constant components of a signal in an amplifier will be calculated below.

It should be stressed particularly that in solving this problem it is not sufficient to calculate the gains for the two waves: it is also necessary to allow for the transfer of energy from one wave to the other, which occurs because of the nonlinearity. Another important aspect which must be allowed for is the appearance in a nonlinear medium of "extra" fields whose transverse structure does not agree with the constant or alternating components, and which will be regarded as interference (noise).

2. DERIVATION OF EQUATIONS

Let us assume that a spatially modulated wave $\mathscr{E}(x,y,z)\exp[ik(z-ct)]$ is traveling along the Z direction in an amplifier. The characteristic scale of the transverse modulation will be denoted by l and it will be assumed that l obeys the condition

$$kl \gg 1. \tag{1}$$

A parabolic equation for the field $\mathscr{E}$ can be written in the form

$$\hat{L}\mathscr{E} \equiv \left[\frac{\partial}{\partial z} + \frac{1}{2ik}\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right)\right]\mathscr{E} = P, \tag{2}$$

$$P = \beta\mathscr{E}/(1+\mathscr{E}\mathscr{E}^*), \tag{3}$$

on the assumption that the medium being investigated is of the two-level type causing homogeneous broadening, and the field will be expressed in units which determine saturation. We shall assume that the medium occupies a region of dimensions $L_x \times L_y \times L$. At the entry to the medium, where $z=0$, the field is

$$\mathscr{E}(x,y,0) = \sqrt{a(0)} + \sqrt{\xi(x,y,0)}\exp[i\varphi(x,y,0)], \tag{4}$$

where $a(0)$ is independent of the transverse coordinates and the complex function of the transverse coordinates $\sqrt{\xi(x,y,0}\exp[i\varphi(x,y,0)]$ has a fairly complicated structure and satisfies the conditions

$$\int_0^{L_x}\int_0^{L_y} dxdy\sqrt{\xi}\exp(i\varphi) = 0, \quad \frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y} dxdy\,\xi = b(0) \tag{5}$$

and belongs to a set of functions whose values for any x, y obey the distribution

$$w(\varphi)\,d\varphi = \frac{1}{2\pi}d\varphi, \quad w(\xi)\,d\xi = \exp\left[-\frac{\xi}{b(0)}\right]\frac{d\xi}{b(0)}. \tag{6}$$

We shall seek the solution of Eq. (2) subject to the boundary condition (4) in the form

$$\mathscr{E} = \mathscr{E}_0 + \mathscr{E}_1, \tag{7}$$

$$\mathscr{E}_0 = \sqrt{a(z)} + \sqrt{\xi(x,y,z)}\exp[i\varphi(x,y,z)], \tag{8}$$

where $\sqrt{\xi}\exp(i\varphi)$ is a function of x,y, and z proportional in each z section to the field expected in the case of propagation of the alternating part of the input signal in free space. For convenience, we shall denote the proportionality factor by $\sqrt{b(z)/b(0)}$; thus, the function $\sqrt{b(0)/b(z)}\times\sqrt{\xi}\exp(i\varphi)$ obeys the following homogeneous equation:

$$\hat{L}\{\sqrt{b(0)/b(z)}\sqrt{\xi(x,y,z)}\exp[i\varphi(x,y,z)]\} = 0. \tag{9}$$

The function $\mathscr{E}_1$ has to satisfy the requirement that in each section z it is orthogonal to the constant component and to the function $\sqrt{\xi}\exp(i\varphi)$.

We have thus separated the two parts in the required solution: the signal (field $\mathscr{E}_0$) and interference (field $\mathscr{E}_1$). The signal contains two fundamentally differing terms: a field constant over the transverse cross section [plane wave of amplitude $\sqrt{a(z)}$] and a component varying rapidly over this cross section [$\sqrt{\xi}\exp(i\varphi)$].

We shall write down the equations for the constant and alternating components of the signal field by introducing projections of the right-hand side of Eq. (2) on these components:

$$\left.\begin{aligned} P_a &= \frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y} dxdyP \equiv \chi_a\sqrt{a}, \\ P_b &= \sqrt{\frac{\xi}{b}}\exp(i\varphi)\frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y} dxdyP\sqrt{\frac{\xi}{b}} \\ &\times\exp(-i\varphi) \equiv \sqrt{\frac{\xi}{b}}\exp(i\varphi)\chi_b. \end{aligned}\right\} \tag{10}$$

Using the above notation, we have

$$\hat{L}(\sqrt{a}) = P_a, \quad \hat{L}(\sqrt{\xi}\exp(i\varphi)) = P_b. \tag{11}$$

Using Eq. (9), and also the fact that a and b depend only on one coordinate z, we find from Eq. (11)

$$\frac{d}{dz}\sqrt{a} = \chi_a\sqrt{a}, \quad \frac{d}{dz}\sqrt{b} = \chi_b\sqrt{b}. \tag{12}$$

We shall now calculate χ_a and χ_b. We shall ignore the influence of interference (exactly as in Ref. 3) and substitute in Eq. (3) the expression for the signal field (8); this gives

$$\left.\begin{aligned} \chi_a &= \beta\frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y} dxdy\frac{\sqrt{a}+\sqrt{\xi}\exp(i\varphi)}{1+a+\xi+2\sqrt{a\xi}\cos\varphi}\frac{1}{\sqrt{a}}, \\ \chi_b &= \beta\frac{1}{L_xL_y}\int_0^{L_x}\int_0^{L_y} dxdy\frac{\sqrt{a}+\sqrt{\xi}\exp(i\varphi)}{1+a+\xi+2\sqrt{a\xi}\cos\varphi}\sqrt{\xi}\exp(-i\varphi)\frac{1}{b}. \end{aligned}\right\} \tag{13}$$

Next, we shall replace in Eq. (13) the averaging over the coordinates with the averaging over the set [see Eq. (6)], which gives

$$\begin{aligned} \chi_a &= \beta\int_0^\infty\frac{d\xi}{b}e^{-\xi/b}\int_0^{2\pi}\frac{d\varphi}{2\pi}\frac{\sqrt{a}+\sqrt{\xi}\exp(i\varphi)}{1+a+\xi+2\sqrt{a\xi}\cos\varphi}\frac{1}{\sqrt{a}}, \\ \chi_b &= \beta\int_0^\infty\frac{d\xi}{b}e^{-\xi/b}\int_0^{2\pi}\frac{d\varphi}{2\pi}\frac{\sqrt{a\xi}+\xi}{1+a+\xi+2\sqrt{a\xi}\cos\varphi}\frac{1}{b}. \end{aligned} \tag{14}$$

Calculating the integrals with respect to φ, we shall reduce the system (14) to

$$\left.\begin{aligned} \chi_a &= \beta \int_0^\infty \frac{d\xi}{a} e^{-\xi/b}\left[1+\frac{a-\xi-1}{\sqrt{(1+a+\xi)^2-4a\xi}}\right] \\ &\times\frac{1}{2}\frac{1}{a} \equiv \beta\frac{1}{2}\left\langle 1+\frac{a-\xi-1}{\sqrt{(1+a+\xi)^2-4a\xi}}\right\rangle, \\ \chi_b &= \beta \int_0^\infty \frac{d\xi}{b} e^{-\xi/b}\left[1-\frac{a-\xi+1}{\sqrt{(1+a+\xi)^2-4a\xi}}\right] \\ &\times\frac{1}{2}\frac{1}{b} \equiv \beta\frac{1}{2}\left\langle 1-\frac{a-\xi+1}{\sqrt{(1+a+\xi)^2-4a\xi}}\right\rangle. \end{aligned}\right\} \tag{15}$$

Averaging over the distribution (6) will be denoted by angular brackets. In terms of the above notation, we finally obtain the following system of coupled equations for the two components of the signal field:

$$\left.\begin{aligned} \frac{d}{dz}\sqrt{a} &= \beta\frac{1}{2}\left\langle 1+\frac{a-\xi-1}{\sqrt{(1+a+\xi)^2-4a\xi}}\right\rangle\frac{1}{\sqrt{a}}, \\ \frac{d}{dz}\sqrt{b} &= \beta\frac{1}{2}\left\langle 1-\frac{a-\xi+1}{\sqrt{(1+a+\xi)^2-4a\xi}}\right\rangle\frac{1}{\sqrt{b}}. \end{aligned}\right\} \tag{16}$$

The equation for the interference field will be found by turning back to Eq. (22) describing the total field $\mathscr{E}_0 + \mathscr{E}_1$:

$$\hat{L}(\mathscr{E}_0+\mathscr{E}_1)=\beta(\mathscr{E}_0+\mathscr{E}_1)/(1+|\mathscr{E}_0+\mathscr{E}_1|^2). \tag{17}$$

Expanding the right-hand side of Eq. (17) in terms of the weak field $\mathscr{E}_1$, we obtain

$$\hat{L}(\mathscr{E}_0+\mathscr{E}_1)=\beta\frac{\mathscr{E}_0}{1+\mathscr{E}_0\mathscr{E}_0^*}+\beta\frac{\mathscr{E}_1}{(1+\mathscr{E}_0\mathscr{E}_0^*)^2}-\beta\frac{\mathscr{E}_1^*\mathscr{E}_0\mathscr{E}_0}{(1-\mathscr{E}_0\mathscr{E}_0^*)^2}. \tag{18}$$

We shall assume that the interference can be estimated using the coefficients in front of $\mathscr{E}_1$ and $\mathscr{E}_1^*$ averaged over the transverse cross section. If we again replace averaging over the coordinates with the averaging over the set, we find that the coefficient in front of $\mathscr{E}_1^*$ vanishes and that in front of $\mathscr{E}_1$ becomes

$$\begin{aligned} \chi &= \left\langle\frac{1}{(1+\mathscr{E}_0\mathscr{E}_0^*)^2}\right\rangle = \left\langle\frac{1}{(1+a+\xi+2\sqrt{a\xi}\cos\varphi)^2}\right\rangle \\ &= \left\langle\frac{1+a+\xi}{[(1+a+\xi)^2-4a\xi]^{3/2}}\right\rangle. \end{aligned} \tag{19}$$

Bearing this point in mind, we obtain the following equation for the interference from Eqs. (7), (16), and (18):

$$\hat{L}\mathscr{E}_1-\chi\mathscr{E}_1=[P-P_a-P_b]\big|_{\mathscr{E}=\mathscr{E}_0}\equiv P_1. \tag{20}$$

3. INTERFERENCE INTENSITY

We can estimate the relative level of the interference in the system under discussion by considering the total radiation flux

$$W=\int_0^{L_x}\int_0^{L_y}\mathscr{E}\mathscr{E}^*\,dxdy. \tag{21}$$

We can easily see that in view of the orthogonality of the functions $\sqrt{a}$, $\sqrt{\xi}\exp(i\varphi)$, and P_1, both components of the signal field and the interference field make additive contributions to the flux:

$$W=W_a+W_b+W_1, \tag{22}$$

where

$$W_a=L_xL_ya; \tag{23}$$

$$W_b=L_xL_yb; \tag{24}$$

$$W_1=\int_0^{L_x}\int_0^{L_y}\mathscr{E}_1\mathscr{E}_1^*\,dxdy. \tag{25}$$

We shall now calculate the flux W_1 in the far-field zone. We shall express the field $\mathscr{E}_1$ in terms of the right-hand side of Eq. (20) using the source function of the parabolic equation, and then we shall substitute this expression in Eq. (25) and carry out transformations similar to those in Ref. 3. This gives

$$\begin{aligned} W_1(z\to\infty) &= \exp\left(2\int_0^L\chi\,d\zeta\right)\frac{ik}{2\pi}\int_0^{L_x}\int_0^{L_y}\int_0^L dxdydz \\ &\times\int_0^{L_x}\int_0^{L_y}\int_0^L dx'dy'dz'\frac{1}{z-z'}\exp\left[-\frac{ik}{2}\frac{(x-x')^2+(y-y')^2}{z-z'}\right] \\ &\times P_1(x,y,z)\exp\left(-\int_0^z\chi\,d\zeta\right)P_1^*(x',y',z')\exp\left(-\int_0^{z'}\chi\,d\zeta\right). \end{aligned} \tag{26}$$

We shall find it convenient to use the approach employed in Ref. 3 and use the Fourier components of the quantity $P_1(x,y,z)\exp(-\int_0^z\chi d\zeta)$. As in Ref. 3, we shall assume that the intensities of all the Fourier components are the same when the wave vectors obey $|k_x|<1/l$, $|k_y|<1/l$, and $|k_z|<1/2kl^2$, but vanish outside these ranges. Repeating the estimates represented by Eqs. (25)–(27) from Eq. (3) (χ_0 should be replaced with χ and $\chi_1\mathscr{E}_0$ with P_1), which gives

$$\begin{aligned} W_1 &= \exp\left(2\int_0^L\chi\,d\zeta\right)L_xL_y\int_0^L\langle P_1P_1^*\rangle\exp\left(-2\int_0^z\chi\,d\zeta\right)dz \\ &\times 2kl^2\,\mathrm{Si}\,(L/2kl^2) \end{aligned} \tag{27}$$

(here, Si is the integral sine). The flux W_1 associated with the interference should be compared with the energy flux of the main field $W_0=W_a+W_b$. In the far-field zone the fluxes W_a and W_b can be expressed in terms of the intensities of the constant and alternating components of the signal at the exit from the amplifier:

$$W_aMW_b=L_xL_y[a(L)+b(L)]; \tag{28}$$

instead of Eq. (28), it is convenient to use the following formal equality:

$$W_a+W_b=L_xL_y[a(z)+b(z)]\exp\left(2\int_z^L\chi_{a+b}d\zeta\right), \tag{29}$$

where $\chi_{a+b}=(\chi_aa+\chi_bb)/(a+b)$. This equality is based on a consequence of Eq. (12), which is $d(a+b)/dz=2(\chi_aa+\chi_bb)$. Combining Eqs. (27) and (29), we obtain

$$\frac{W_1}{W_0}=2kl^2\,\mathrm{Si}\left(\frac{L}{2kl^2}\right)\int_0^L\frac{\langle P_1P_1^*\rangle}{a+b}\left[\exp 2\int_0^L(\chi-\chi_{a+b})\,d\zeta\right]dz. \tag{30}$$

We recall that in the case discussed in Ref. 3, when the signal field has no constant component, the signal and interference gains are equal ($\chi=\chi_0$, $\chi_{a+b}=\chi_0$) and the argument of the exponential function vanishes in the formula for the ratio W_1/W_0. On the other hand, if there is no alternating component, we have $\chi_{a+b}=1/(1+a)$ and $\chi=1/(1+a)^2$, i.e., $\chi<\chi_{a+b}$.

In the general case ($a\neq 0$, $b\neq 0$) one would need to carry out numerical calculations. Such calculations demonstrate that the inequality $\chi<\chi_{a+b}$ is always obeyed (only for $a=0$ it reduces to $\chi=\chi_{a+b}$). Bearing this point in mind, we shall replace Eq. (30) with the inequality

$$\frac{W_1}{W_0}\leqslant 2kl^2\,\mathrm{Si}\frac{L}{2kl^2}\int_0^L\frac{\langle P_1P_1^*\rangle}{a+b}dz. \tag{31}$$

We can estimate the ratio $\langle P_1P_1^*\rangle/(a+b)$ if we can express $\langle P_1P_1^*\rangle$ in terms of a and b. Averaging gives

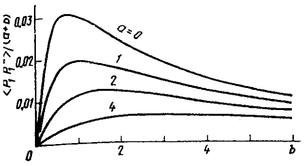

FIG. 1. Dependence of $\langle P_1 P_2^* \rangle/(a+b)$ on b.

$$\langle P_1 P_1^* \rangle = \langle |P - P_a - P_b|^2 \rangle = \langle PP^* \rangle - \chi_a^2 a - \chi_b^2 b$$
$$= \langle 1/\sqrt{(1+a+\xi)^2 - 4a\xi} \rangle - \langle (1+a+\xi)/[(1+a+\xi)^2 - 4a\xi]^{3/2} \rangle$$
$$- {}^1/_4 \langle 1 + (a - \xi - 1)/\sqrt{(1+a+\xi)^2 - 4a\xi} \rangle^2 / a$$
$$- {}^1/_4 \langle 1 - (a - \xi + 1)/\sqrt{(1+a+\xi)^2 - 4a\xi} \rangle^2 / b. \quad (32)$$

Numerical calculations were carried out on the basis of Eq. (32). The results of these calculations are presented in Fig. 1 where the curves representing $\langle P_1 P_1^* \rangle/(a+b)$ are plotted.

The maximum value of $\langle P_1 P_1^* \rangle(a+b)$ occurs at $a = 0$ and $b = 0.8$ and it amounts to $0.03\beta^2$. Thus, Eq. (31) can be replaced with

$$W_1/W_0 < 0{,}03\beta^2 L 2kl^2\, \mathrm{Si}(L/2kl^2). \quad (33)$$

We can now see that the radiation flux representing interference is much weaker than the output signal if the value of βL is moderate. However, the ratio of these two quantities decreases considerably if diffraction effects occur. In fact, if the amplifying medium is short (i.e., when $L \ll 2kl^2$) and there is no diffraction smearing of the signal structure in the amplifier, we find that $2kl^2 L^{-1}\,\mathrm{Si}(L/2kl^2) = 1$ and Eq. (33) gives $W_1/W_0 < 0.03\,(\beta/L)^2$. When $L/2kl^2$ increases, the quantity $2kl^2 L^{-1}\,\mathrm{Si}\,(L/2kl^2)$ decreases and for $L/2kl^2 \gg 1$ it can be replaced with a small quantity $\pi k l^2/L$, which gives $W_1/W_0 \leq 0.05\,(\beta l)^2 2kl^2/L$. It should be stressed that this estimate of the interference given by Eq. (33) is grossly overestimated. More accurate estimates can be obtained only if more detailed numerical calculations are made. The smallness of the ratio W_1/W_0 confirms the validity of the splitting of the total field into the main or signal field $\mathscr{E}_0 = \sqrt{a} + \sqrt{\xi}\exp(i\varphi)$ and the weak interference field $\mathscr{E}_1$.

4. CHANGES IN THE INTENSITIES OF THE CONSTANT AND ALTERNATING COMPONENTS OF A SIGNAL IN AN AMPLIFIER

In view of the smallness of the interference, the signal field can be considered independently of the interference, i.e., we can describe the signal by the system (12). It is convenient to modify the system (12) by replacing the coefficients $\sqrt{a}$ and $\sqrt{b}$ of the constant and variable components of the signal field with their squares a and b. It then follows from Eqs. (12) and (15) that

$$\frac{da}{d(2\beta z)} = \frac{1}{2}\left(1 + \frac{a - \xi - 1}{\sqrt{(1+a+\xi)^2 - 4a\xi}}\right),$$
$$\frac{db}{d(2\beta z)} = \frac{1}{2}\left(1 - \frac{a - \xi + 1}{\sqrt{(1+a+\xi)^2 - 4a\xi}}\right). \quad (34)$$

It should be noted that at low intensities we readily obtain from the system (34)

$$\frac{da}{d(2\beta z)} = a, \quad \frac{db}{d(2\beta z)} = b. \quad (35)$$

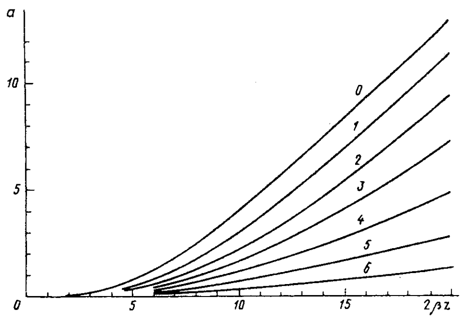

FIG. 2. Dependences of the intensity of the constant component a of a signal on the amplifier length for different input parameters: 0) $a = 0.01$, $b = 0$; 1) $a = 0.0067$, $b = 0.0033$; 2) $a = 0.005$, $b = 0.005$; 3) $a = 0.004$, $b = 0.006$; 4) $a = 0.003$, $b = 0.007$; 5) $a = 0.002$, $b = 0.008$; 6) $a = 0.001$, $b = 0.009$.

The system (35) states the obvious result: in the linear regime both components increase exponentially with the same growth increments. In the other limiting case when $a \gg 1$ and $b \gg 1$, we obtain from Eq. (34)

$$\frac{da}{d(2\beta z)} = 1 - e^{-a/b}, \quad \frac{db}{d(2\beta z)} = e^{-a/b}. \quad (36)$$

It follows from Eq. (36) that in the case of strong saturation the total field energy increases linearly, $d/(a+b)/d(2\beta z) = 1$, but the constant component is amplified preferentially. In fact, it follows from Eq. (36) that

$$\frac{d(a/b)}{d(2\beta z)} = \frac{e^{-a/b}}{b}\left(e^{a/b} - 1 - \frac{a}{b}\right) > 0.$$

Thus, the ratio a/b must increase when the saturation is strong. If the amplifier is sufficiently long, then beginning from a certain coordinate z the value of a/b becomes so large that Eq. (36) changes to

$$\frac{da}{d(2\beta z)} = 1, \quad \frac{db}{d(2\beta z)} = 0. \quad (37)$$

It follows from the system (37) that in the case of very considerable amplification lengths the regime is such that the constant component of the signal increases linearly with the coordinate and the energy stored in the alternating component reaches a saturation value and does not increase any further. At moderate saturations, which are of greatest practical importance in the case of image amplifiers, one has to solve numerically the system (34). We obtained such solutions for a series of input data. The initial values were selected so that the power of the whole signal at the input was the

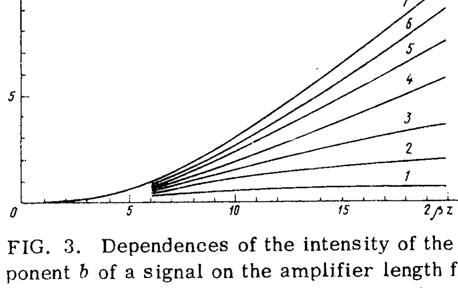

FIG. 3. Dependences of the intensity of the alternating component b of a signal on the amplifier length for different input parameters: 1) $a = 0.0067$, $b = 0.0033$; 2) $a = 0.005$, $b = 0.005$; 3) $a = 0.004$, $b = 0.006$; 4) $a = 0.003$, $b = 0.007$; 5) $a = 0.002$, $b = 0.008$; 6) $a = 0.001$, $b = 0.009$; 7) $a = 0$, $b = 0.01$.

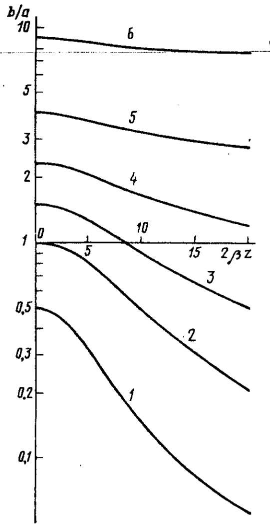

FIG. 4. Dependences of the depth of modulation of a signal on the amplifier length for the following input parameters: 1) $a=0.0067$, $b=0.0033$; 2) $a=0.005$, $b=0.005$; 3) $a=0.004$, $b=0.006$; 4) $a=0.003$, $b=0.007$; 5) $a=0.002$, $b=0.008$; 6) $a=0.001$, $b=0.009$.

same in all cases: the condition $a(0)+b(0)=0.01$ was obeyed in all cases. In view of the invariance of the equations in the system (34) relative to the shift of z, the results of the calculations could be used also in the case when higher intensities were assumed at the input. The results of these calculations are plotted in Figs. 2 and 3. The uppermost curve in Fig. 2 represents the case of propagation of a plane wave in an amplifier, i.e., it corresponds to the absence of any modulation: $da/d(2\beta z)=a/(1+a)$, $b=0$. The uppermost curve in Fig. 3 represents the opposite case, when the modulation is total, i.e., when a constant component is absent:

$$a=0, \frac{db}{d(2\beta z)}-1+\frac{1}{b}\left(\exp\frac{1}{b}\right)\mathrm{Ei}\left(-\frac{1}{b}\right) \tag{38}$$

(Ei is the exponential integral). Equation (38) was obtained in Ref. 3. It is interesting to note curves 1 and 2 in Fig. 3, which reveal that the alternating component has already reached saturation. We can also see from the two figures that the constant component increases faster in the intensity than the alternating one. For example, assuming the input values $a(0)=0.004$ and $b(0)=0.006$, we find that for $2\beta z=8.4$ the constant component becomes greater than the alternating component (curve 3 in Figs. 2 and 3).

Figure 4 shows how the ratio b/a varies with the coordinate. We can see that in all the cases discussed above the depth of modulation of the signal (i.e., the quantity b/a) decreases significantly as a result of amplification. Only in the case of the largest (of those given in Fig. 4) input value of $b/a=9$ is the change in the depth of modulation not too great. This means that for this input signal and total gain $2\beta L=20$ the distortions of the signal at the amplifier exit are slight. Reduction in the input value of b/a enhances the importance of distortions of the signal in the amplifier.

We shall conclude by stressing that the approach adopted above has made it possible to reduce the equation for a nonlinear medium expressed in terms of partial derivatives to a system of two ordinary differential equations, and to obtain a solution in a clear form. This method should be convenient in analyzing other problems involving quantum amplifiers.

We have shown above that the most important distortions occurring in an amplifier in the course of propagation of a spatially modulated wave with a pseudorandom transverse structure involve reduction in the depth of modulation. Such distortions can easily be removed by filtering a Fourier plane. The results obtained allow us to estimate the reduction in the depth of modulation which can be expected under specific conditions.

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