Interaction between counterpropagating spatially modulated beams in a nonlinear medium

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An investigation is made of the resonant amplification of two counterpropagating monochromatic beams having pseudorandom spatial modulation. A system of equations describing the change in the average intensities of the beams in the direction of propagation is derived and solved.

Optical signals having a pseudorandom transverse structure propagating through a nonlinear (amplifying) medium were investigated in Refs. 1 and 2 for waves traveling in one direction. The case when counterpropagating waves travel in a medium is considered below. The presence of transverse modulation complicates considerably the interaction, compared with the case of plane waves considered in Refs. 3 and 4. The situation is simplified slightly if the wave modulation is complete, i.e., if both waves have a zero average over the transverse cross section. The differential equation for the average intensities of the counterpropagating waves can then be solved in terms of quadratures.

We shall analyze a field of the type $Ee^{-i\omega t}$. We shall assume that E satisfies the equation

$$(\Delta+k^2)E=ik^2\,(\delta\varepsilon/\varepsilon_0)E, \tag{1}$$

where $k^2=\varepsilon_0\omega^2/c^2$; $\varepsilon=\varepsilon_0-i\delta\varepsilon$ is the permittivity of the medium; $\delta\varepsilon=2\beta c/\omega(1+E^*E)$. This dependence corresponds to a two-level homogeneously broadened medium. We shall assume that $\delta\varepsilon\ll 1$.

In order to isolate explicitly the counterpropagating fields, we shall write $E=\mathscr{E}_1e^{ikz}+\mathscr{E}_2e^{-ikz}$, where $\mathscr{E}_1$ and $\mathscr{E}_2$ are slowly varying functions of the coordinates. Converting to a parabolic approximation, we obtain

$$e^{ikz}\left(2ik\frac{d}{dz}+\Delta_\perp\right)\mathscr{E}_1+e^{-ikz}\left(-2ik\frac{d}{dz}+\Delta_\perp\right)\mathscr{E}_2 = ik2\beta\frac{\mathscr{E}_1e^{ikz}+\mathscr{E}_2e^{-ikz}}{1+|\mathscr{E}_1e^{ikz}+\mathscr{E}_2e^{-ikz}|}. \tag{2}$$

Multiplying Eq. (2) by $e^{-ikz}/(2ik)$ and averaging over z between $z-\pi/2k$ and $z+\pi/2k$, we obtain

$$\left(\frac{d}{dz}+\frac{1}{2ik}\Delta_\perp\right)\mathscr{E}_1=\frac{\beta}{2\pi}\int_{-\pi}^{\pi}d\,(2kz)\,\frac{\mathscr{E}_1+\mathscr{E}_2e^{-2ikz}}{1+|\mathscr{E}_1+\mathscr{E}_2e^{-2ikz}|}. \tag{3}$$

An equation for the counterpropagating wave is derived similarly:

$$\left(\frac{d}{dz}-\frac{1}{2ik}\Delta_\perp\right)\mathscr{E}_2=-\frac{\beta}{2\pi}\int_{-\pi}^{\pi}d\,(2kz)\,\frac{\mathscr{E}_2+\mathscr{E}_1e^{2ikz}}{1+|\mathscr{E}_2+\mathscr{E}_1e^{2ikz}|}. \tag{4}$$

We recall the problem of interacting plane waves. For functions $\mathscr{E}_1$ and $\mathscr{E}_2$ that do not depend on the transverse coordinates the system (3) and (4) yields the familiar system of ordinary differential equations

$$\frac{d}{dz}\mathscr{E}_1=\beta F(\mathscr{E}_1\mathscr{E}_1^*,\ \mathscr{E}_2\mathscr{E}_2^*)\,\mathscr{E}_1,\quad \frac{d}{dz}\mathscr{E}_2=-\beta F(\mathscr{E}_2\mathscr{E}_2^*,\ \mathscr{E}_1\mathscr{E}_1^*)\,\mathscr{E}_2, \tag{5}$$

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where F is an elementary function of two real variables:

$$F(x,\ y)=\left[1-(1-x+y)/\sqrt{1+2(x+y)+(x-y)^2}\right]/2x. \quad (6)$$

For simplicity, it can be assumed that $\mathscr{E}_1$ and $\mathscr{E}_2$ are real. In this case, the general solution of the system (5) takes the form

$$1+\mathscr{E}_1^2+\mathscr{E}_2^2-\sqrt{\left(1+\mathscr{E}_1^2+\mathscr{E}_2^2\right)^2-4\mathscr{E}_1^2\mathscr{E}_2^2}=2I_{\min}=\text{const}, \quad (7a)$$

$$\mathscr{E}_2^2-\mathscr{E}_1^2+\ln\frac{\left(\mathscr{E}_1^2+\mathscr{E}_2^2\right)\left(\sqrt{1+(\mathscr{E}_1-\mathscr{E}_2)^2}+1\right)-\left(\mathscr{E}_1^2-\mathscr{E}_2^2\right)\left(\sqrt{1+(\mathscr{E}_1-\mathscr{E}_2)^2}+1\right)}{\left(\mathscr{E}_1^2+\mathscr{E}_2^2\right)\left(\sqrt{1+(\mathscr{E}_1-\mathscr{E}_2)^2}+1\right)+\left(\mathscr{E}_1^2-\mathscr{E}_2^2\right)\left(\sqrt{1+(\mathscr{E}_1-\mathscr{E}_2)^2}+1\right)}-2\beta z=2\beta z_0=\text{const}. \quad (7b)$$

The physical meaning of the constants $I_{\min}$ and z_0 is clear from Fig. 1.

We are interested in the case when $\mathscr{E}_1$ and $\mathscr{E}_2$ are waves having a complex structure, a zero average, and different transverse modulations. The interaction between one such wave and a nonlinear medium is well known.[1]

We shall seek a solution of the system (3) and (4) in the form

$$\mathscr{E}_1=B_1f_1+\mathscr{F}_1,\quad \mathscr{E}_2=B_2f_2+\mathscr{F}_2, \quad (8)$$

where $B_{1,2}$ only depends on z; $f_{1,2}$ are the solutions of equations for free space;

$$\frac{d}{dz}f_1+\frac{1}{2ik}\Delta_\perp f_1=0,\quad \frac{d}{dz}f_2-\frac{1}{2ik}\Delta_\perp f_2=0; \quad (9)$$

$\mathscr{F}_{1,2}$ are relatively weak fields that are logically described as noise. It is important to note that for the case of completely modulated waves there are no conjugate fields. If there were sources of conjugate fields (and these certainly do occur in the presence of a constant component), these fields would be weak and could not be included in the noise. For this reason the absence of a constant component considerably simplifies the analysis.

We shall assume that the solutions of the equations (9) are normalized: $\langle f_1f_1^*\rangle_\perp=\langle f_2f_2^*\rangle_\perp=1$, where $\langle\ldots\rangle_\perp$ denotes averaging over the transverse coordinates. Substituting Eq. (8) into the system (3) and (4) and allowing for Eq. (9), we obtain

$$\frac{dB_1}{dz}f_1+\frac{d\mathscr{F}_1}{dz}+\frac{1}{2ik}\Delta_\perp\mathscr{F}_1=\frac{\beta}{2\pi}\int_{-\pi}^{\pi}d(2kz)\frac{\mathscr{E}_1+\mathscr{E}_2e^{-i2kz}}{1+|\mathscr{E}_1+\mathscr{E}_2e^{-i2kz}|^2}, \quad (10a)$$

$$\frac{dB_2}{dz}f_2+\frac{d\mathscr{F}_2}{dz}-\frac{1}{2ik}\Delta_\perp\mathscr{F}_2=\frac{-\beta}{2\pi}\int_{-\pi}^{\pi}d(2kz)\frac{\mathscr{E}_2+\mathscr{E}_1e^{i2kz}}{1+|\mathscr{E}_2+\mathscr{E}_1e^{i2kz}|^2}. \quad (10b)$$

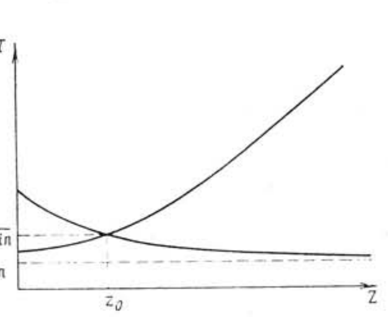

FIG. 1. Dependences of the intensity of counterpropagating plane waves on the coordinate z.

The integrals in the system (10) can be calculated for and then the right-hand sides of the equations (10) can be expressed in terms of the function F given by Eq. (6), but this does not simplify the following calculations.

Finding the projections of the right-hand sides of the system (10) on the fields f_1, f_2, we write

$$\frac{d}{dz}B_1=\beta\left\langle\frac{1}{2\pi}\int_{-\pi}^{\pi}d(2kz)\frac{\mathscr{E}_1+\mathscr{E}_2e^{-i2kz}}{1+|\mathscr{E}_1+\mathscr{E}_2e^{-i2kz}|^2}f_1^*\right\rangle_\perp=\chi_1B_1, \quad (11a)$$

$$\frac{d}{dz}B_2=-\beta\left\langle\frac{1}{2\pi}\int_{-\pi}^{\pi}d(2kz)\frac{\mathscr{E}_2+\mathscr{E}_1e^{i2kz}}{1+|\mathscr{E}_2+\mathscr{E}_1e^{-i2kz}|^2}f_2^*\right\rangle_\perp=-\chi_2B_2. \quad (11b)$$

Substituting the system (11) into the system (10), we obtain an equation for the noise:

$$\left(\frac{d}{dz}+\frac{1}{2ik}\Delta_\perp\right)\mathscr{F}_1=\frac{\beta}{2\pi}\int_{-\pi}^{\pi}d(2kz)\frac{\mathscr{E}_1+\mathscr{E}_2e^{-i2kz}}{1+|\mathscr{E}_1+\mathscr{E}_2e^{-i2kz}|^2}-\chi_1B_1f_1, \quad (12a)$$

$$\left(-\frac{d}{dz}+\frac{1}{2ik}\Delta_\perp\right)\mathscr{F}_1=\frac{\beta}{2\pi}\int_{-\pi}^{\pi}d(2kz)\frac{\mathscr{E}_2+\mathscr{E}_1e^{i2kz}}{1+|\mathscr{E}_2+\mathscr{E}_1e^{i2kz}|^2}-\chi_2B_2f_2. \quad (12b)$$

We shall solve the system of equations (11) for the fundamental-frequency fields using the substitution $\mathscr{F}_1=\mathscr{F}_2=0$, which is equivalent to assuming that the noise is relatively weak.

As in Refs. 1 and 2, we shall modify the system (11) by replacing the averaging over the transverse cross section with the averaging over an ensemble. Assuming that the fields carry images of complex objects, we shall assume that each of these is described by a Gaussian distribution having a zero average. We shall also assume that these fields are independent. Performing the substitution

$$\langle\Phi\rangle_\perp\to\langle\Phi\rangle_{f_1,f_2}\equiv\frac{1}{\pi^2}\int d^2f_1d^2f_2e^{-f_1f_1^*-f_2f_2^*}\Phi, \quad (13)$$

for $\chi_{1,2}$ from the system (11) we obtain

$$\chi_1=\beta\frac{1}{2\pi}\int_{-\pi}^{\pi}d(2kz)\left\langle\frac{B_1f_1+B_2f_2e^{-i2kz}}{1+|B_1f_1+B_2f_2e^{-i2kz}|^2}\frac{f_1^*}{B_1}\right\rangle_{f_1,f_2}, \quad (14a)$$

$$\chi_2=\beta\frac{1}{2\pi}\int_{-\pi}^{\pi}d(2kz)\left\langle\frac{B_2f_2+B_1f_1e^{i2kz}}{1+|B_2f_2+B_1f_1e^{i2kz}|^2}\frac{f_2^*}{B_2}\right\rangle_{f_1,f_2}. \quad (14b)$$

In this case, integration with respect to $2kz$ is removed from the integrals with respect to f_1 and f_2. The integrals in the system (14) are taken by substituting

$$f_1 = B_1^* v_1 / \sqrt{B_1^* B_1 + B_2 B_2^*} - B_2 v_2 / \sqrt{B_1 B_1^* + B_2 B_2^*}, \quad (15a)$$

$$f_2 e^{-i2kz} = B_2^* v_1 + \sqrt{B_1 B_1^* + B_2 B_2^*} + B_1 v_2 / \sqrt{B_1 B_1^* + B_2 B_2^*}. \quad (15b)$$

As a result, we have

$$\chi_1 = \chi_2 = \beta\chi \left(B_1 B_1^* + B_2 B_2^*\right), \quad (16)$$

where χ is a real function of a single variable expressed in terms of the exponential integral:

$$\chi(b) = \int_0^\infty dx e^{-x} \frac{x}{1+bx} = \frac{1}{b}\left(1 + \frac{e^{1/b}}{b}\,\mathrm{Ei}\left(-\frac{1}{b}\right)\right). \quad (17)$$

Equation (17) was derived earlier[1] for a single modulated wave. Converting to intensities, we obtain the following system of equations

$$\frac{d}{dz} b_1 = 2\beta\chi(b_1 + b_2) b_1, \quad -\frac{d}{dz} b_2 = 2\beta\chi(b_1 + b_2) b_2. \quad (18)$$

The general solution of the system (18) takes the form

$$b_1(z) b_2(z) = I_0^2 = \mathrm{const}, \; 2\beta z - \int_{I_0}^{b_1(z)} \frac{dx}{\left(x + I_0^2/x\right) \chi\left(x + I_0^2/x\right)} = 2\beta z_0 = \mathrm{const}. \quad (19)$$

The constants I_0 and z_0 introduced here have simple meanings: I_0 is the wave intensity at the point z_0 where both waves have the same intensities.

An estimate of the noise intensity is derived from the equations (12) in the same way as for a single modulated wave.[1] Noise having the following intensity is generated in an amplifier of length L:

$$|\mathcal{F}_1(L)|^2 \lesssim \beta^2 L_d \int_0^L dz \Pi(b_1(z) + b_2(z)) \frac{b_1(z)}{b_1(z) + b_2(z)} \times \exp\left[-2\beta \int_z^L dt \chi(b_1(t) + b_2(t))\right] \mathrm{Si}\left(\frac{L}{L_d}\right), \quad (20)$$

where L_d is the diffraction length characterizing the counterpropagating beams and the function Π is uniformly bounded:

$$0 \leqslant \Pi \leqslant (b) = 1 - \chi(b) - b\chi(b)(1 + \chi(b)) \leqslant 0.03. \quad (21)$$

A simpler estimate, that is more approximate than Eq. (20), can be given for the ratio of the noise intensity to the signal intensity:

$$|\mathcal{F}_1(L)|^2 / b_1(L) \lesssim 0{,}05 \; \beta^2 L_d L. \quad (22)$$

Since this ratio is small, it is justifiable to neglect the noise when deriving the system of equations (10) for the fundamental-frequency fields.

Figure 2 illustrates the difference between the interaction of plane waves and the case when the counterpropagating waves have a complex transverse structure by showing the intensities as a function of the longitudinal coordinate z. Since the systems (5) and (11) are translationally invariant, the origin may be placed at any point. The only isolated point is the coordinate where the intensities of the counterpropagating waves are the same. The pattern is symmetrical relative to this point ($z = 0$), so that only half the graph is shown in Fig. 2. The lower curve of each pair corresponds to the wave which is amplified and propagates to the left and the upper curve corresponds to that propagating to the right.

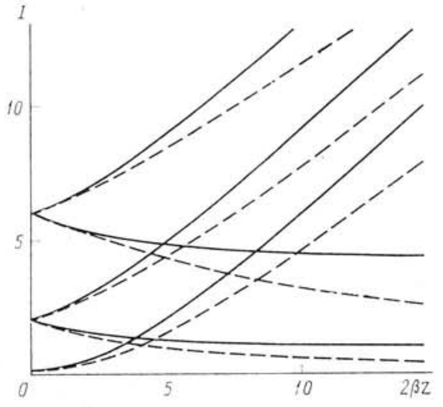

FIG. 2. Dependences of the intensity of counterpropagating plane (continuous curves) and modulated (dashed curves) waves on the coordinate.

It may be noted that in the case of plane waves, the wave that has reached the point $z = 0$ having the fixed intensity I_0 could not be arbitrarily weak at infinity. This wave has an intensity higher than $I_{\min} = I_0 + 0.5 - \sqrt{I_0 + 0.25}$ at all points in the amplifier.

When the counterpropagating waves are modulated, the pattern is different. For a given intensity at the point $z = 0$ the wave entering the amplifier may be extremely weak provided that the amplifier is fairly long.

We shall analyze the influence of this optical beam interaction on the properties of the amplifier in which the counterpropagating waves are propagating. We shall compare the amplification of two plane and two transversely modulated waves.

We shall introduce the characteristic of utilization of the active-medium inversion W which we shall define as the sum of the products of the field intensities and the appropriate increments. The power extracted from the amplifier is defined as the integral of W over the volume. For counterpropagating plane waves, we have

$$W_{\mathrm{pl}} = 2\beta F(x, y) x + 2\beta F(y, x) y = 2\beta\left(1 - 1/\sqrt{1 + 2(x+y) + (x-y)^2}\right), \quad (23)$$

and for completely modulated waves, we have

$$W_m = 2\beta\chi(x+y)x + 2\beta\chi(x+y)y = 2\beta(x+y)\chi(x+y). \quad (24)$$

In Eqs. (23) and (24) x and y are the intensities of the counterpropagating beams and the functions F and χ are given by Eqs. (6) and (17). The quantity $W/2\beta$ may be described as the inversion utilization coefficent. Figure 3 shows dependences of this coefficient on the total intensity $b = x + y$. Curve 1 applies to the case of modulated waves and describes the dependence of the coefficient $W_m/2\beta$. This is extremely similar to the dependence of the coefficient $W/2\beta$ for standing waves:

$$W_{\mathrm{st}}/2\beta = W_{\mathrm{pl}}/2\beta \Big|_{\substack{x+y=b \\ x-y=0}} = 1 - 1/\sqrt{1+2b}.$$

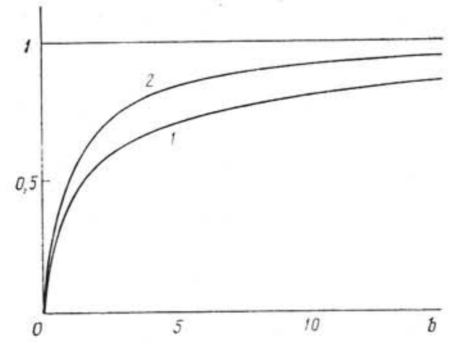

FIG. 3. Dependences of the coefficient of utilization of the inversion $W/2\beta$ on the total intensity b for modulated waves (1) and a plane traveling wave (2).

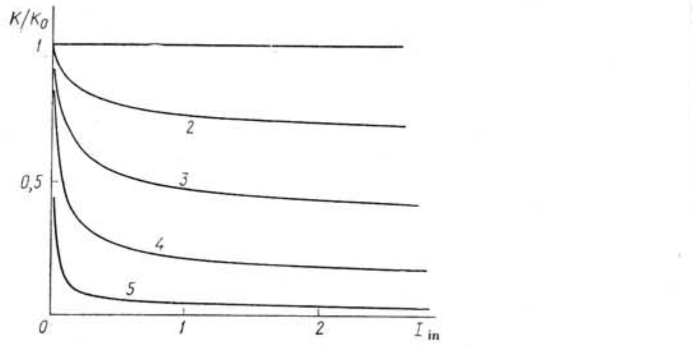

FIG. 4. Dependences of the normalized gain $K/K_0 = I_{out}/e^{4\beta L} I_{in}$ on the input intensity I_{in} for $2\beta L = 0$ (1), 0.2 (2), 0.5 (3), 1 (4), and 2 (5).

Figure 3 does not give $W_{st}/2\beta$ because the relative difference between W_m and W_{st} does not exceed 6%. For intensities in the range $b < 4.6$ a standing wave 'extracts' a somewhat greater power from a unit volume than does a transversely modulated wave, but in the range $b > 4.6$ a modulated wave is somewhat more "productive." For comparison, Fig. 3 shows the intensity dependence of the coefficient $W/2\beta$ for a traveling wave (curve 2):

$$W_{tr}/2\beta = W_{pl}/2\beta\,\big|_{|x-y|=x+y=b} = b/(1+b).$$

The situation when the intensities of the counterpropagating waves differ substantially in an appreciable part of the resonator is of practical interest. In this case, the value of $W/2\beta$ for plane waves will be higher than for modulated waves for any b. A plane wave does not induce a structure in the nonlinear medium and is thus amplified more strongly. This was shown in Refs. 1 and 2 for waves propagating in the same direction and similar topics were discussed recently in Ref. 5.

We shall apply these results to calculations for a double-pass amplifier. We shall assume that after a single pass through an amplifier, a light wave is incident on a complex mirror that alters the transverse structure of the wave but does not dissipate its energy. After the second pass, the wave leaves the system without hindrance. In this case, counterpropagating waves having different transverse structures will propagate in the amplifying medium and these will be described by the system (18). Figure 4 shows the gain $K = I_{out}/I_{in}$ of this amplifier normalized to the weak field gain $K_0 = e^{4\beta L}$ as a function of the input intensity. Similar curves for plane waves in a double-pass amplifier (in this case the mirror should be assumed to be ideal) will resemble those for modulated waves although the intensities on the mirror will differ appreciably (Fig. 2).

In conclusion, we reemphasize that in the case under study the counterpropagating waves travel so that the gain of each wave may be described by an effective increment. The increment is the same for both modulated waves and only depends on the sum of the intensities. The total gain was obtained by numerical calculations for various values of the unsaturated gain.

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