HOW WIDE CAN BE A DISC LASER?

Dmitrii Kouznetsov, Jean-François Bisson, Jun Dong, Ken-ichi Ueda

Institute for Laser Science, University of Electro-Communications, 1-5-1 Chofu, Tokyo, 182-8585, Japan
The scaling of size $L$ increases the Amplified Spontaneous Emission (ASE); the gain $G$ should be reduced. The round-trip gain $g = 2Gh$ should remain larger than the surface scattering losses $\beta$; the thickness $h$ should be increased. At some critical size $L_{\text{max}}$, the overheating does not allow to pump the medium well above the threshold.

The general limit of the power scaling comes not only from the overheating and ASE but also from the surface scattering losses (which are usually believed to be negligible small).
Simple model of laser

Coefficient of surface losses of signal $\beta \ll 1$. Round-trip gain $g \ll 1 \rightarrow$ constant intensity.

Let $\theta$ be output coupling parameter; $g = \theta + \beta$.

It determines the gain $G = g/(2h)$ and the output efficiency $\eta_{\text{output}} = \frac{\theta I_s L^2}{g I_s L^2} = 1 - \frac{\beta}{g}$

Threshold pump power $P_{\text{th}} = (\hbar \omega_p/\tau) L^2 g/(2\sigma)$

Assume that there is no parazitic oscillations.

Amplified Spontaneous Emission (ASE):
Assume that ASE travels a distance $L$ with gain, so, the effective lifetime $\tau = \tau_o \exp(-GL)$.

where gain $G = g/(2h)$.

Then, the threshold pump power

$$P_{th} = QgL^2 \exp\left(\frac{Lg}{2h}\right) ; \quad Q = \frac{\hbar \omega_p}{2\tau_o \sigma}$$
Maximal pump power: \( P_{p,\text{max}} = \frac{RL^2}{h} \)

where \( R = \min \left\{ \frac{3R_T}{\eta_h}, \frac{2k\Delta T_{\text{max}}}{\eta_h} \right\} \),

maximal increase of temperature \( \Delta T \) allowed;

thermal shock parameter \( R_T = \frac{k\sigma_t(1-\nu)}{\alpha E} \)

Output signal power \( P_s = \eta_o \left( 1 - \frac{\beta}{g} \right) \left( P_p - P_{th} \right) \)

What values of \( g \), \( h \), \( L \) are optimal?
Maximize \( P_s = \eta_o \left(1 - \frac{\beta}{g}\right) \left(P_p - Q L^2 g \exp\left(\frac{gL}{2h}\right)\right) \)

at given \( P_p \) as function of \( L, g \); \( h = \frac{RL^2}{P_p} \)

\[
\frac{\partial P_s}{\partial L} = 0 \quad \rightarrow \quad \frac{g P_p}{2RL} = 2 \quad \text{Then} \quad L = \frac{g P_p}{4R}
\]

\[
P_s = \eta_o P_p \left(1 - \frac{\beta}{g}\right) \left(1 - \frac{e^2 Q P_p g^3}{4 R^2}\right).
\]

\[
P_s \approx P_p \eta_o \left(1 - \frac{\beta}{g} - \frac{e^2 Q P_p}{4 R^2} g^3\right)
\]

\[
\text{maximum at} \quad g = \left(\frac{4 \beta R^2}{3e^2 Q P_p}\right)^{1/4}
\]
Maximize $P_s$ at given efficiency $\eta = \frac{P_s}{P_p}$

\[
P_p = \frac{27}{64e^2} \left( \frac{R^2}{Q \beta^3} \right) \left( 1 - \frac{\eta}{\eta_0} \right)^4
\]

\[
P_s(\eta) = \frac{27}{64e^2} \left( \frac{R^2}{Q \beta^3} \right) \left( 1 - \frac{\eta}{\eta_0} \right)^4
\]
Maximal power achievable at given $\beta, R, Q$:

\[
L_{\text{max}} = \frac{9}{8e^2} \frac{R}{Q} \frac{1}{\beta^2}
\]

\[
h_{\text{max}} = \frac{3}{8e^2} \frac{R}{Q} \frac{1}{\beta}
\]

\[
g_{\text{max}} = \frac{4}{3} \beta
\]

\[
P_{p,\text{max}} = \frac{27}{8e^2} \frac{R^2}{Q} \frac{1}{\beta^3}
\]

\[
P_{s,\text{max}} = \frac{27}{64e^2} \frac{R^2}{Q} \frac{\eta_o}{\beta^3}
\]

\[
\frac{P_{s,\text{max}}}{P_{p,\text{max}}} = \frac{\eta_o}{8}
\]

Maximal output power implies low efficiency $\sim 10\%$
Key parameter $P_D = \frac{R^2 \eta_o}{Q \beta^3}$ determines the maximal power of the disk laser.

For the efficient operation ($\eta > 50\%$), the laser should be optimized for power of order of few percents of the maximal power achievable in this configuration.
Example of parameters optimized for given $P_p$
$R = 5 \text{ W/mm}$, $Q = 50 \text{ W/mm}^2$, $\eta_o = 0.91$, $\beta = 0.01$

<table>
<thead>
<tr>
<th>$P_p$, kW</th>
<th>$L$, mm</th>
<th>$h$, mm</th>
<th>$g$</th>
<th>$G$, mm$^{-1}$</th>
<th>$P_s$, kW</th>
<th>$\eta$, %</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.541</td>
<td>0.012</td>
<td>0.031</td>
<td>1.298</td>
<td>0.598</td>
<td>60</td>
</tr>
<tr>
<td>2</td>
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<td>0.017</td>
<td>0.026</td>
<td>0.772</td>
<td>1.082</td>
<td>54</td>
</tr>
<tr>
<td>4</td>
<td>4.359</td>
<td>0.024</td>
<td>0.022</td>
<td>0.459</td>
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<tr>
<td>6</td>
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<td>0.020</td>
<td>0.339</td>
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<tr>
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<td>0.018</td>
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<td>3.157</td>
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<tr>
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<td>0.017</td>
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</tr>
<tr>
<td>228</td>
<td>10.000</td>
<td>0.508</td>
<td>0.013</td>
<td>0.026</td>
<td>26.0</td>
<td>11</td>
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</tbody>
</table>
Example of scaling up $P_{s,\text{max}}$ reducing $\beta$:

$R = 5 \text{ W/mm}$, $Q = 50 \text{ W/mm}^2$, $\eta_o = 0.91$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$L$, mm</th>
<th>$h$, mm</th>
<th>$g$</th>
<th>$G$, mm$^{-1}$</th>
<th>$P_{p,\text{max}}$, kW</th>
<th>$P_{s,\text{max}}$, kW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.002</td>
<td>3806</td>
<td>2.538</td>
<td>0.003</td>
<td>0.001</td>
<td>28547.286</td>
<td>3247.254</td>
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<tr>
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<td>0.005</td>
<td>0.004</td>
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<tr>
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<td>152</td>
<td>0.508</td>
<td>0.013</td>
<td>0.026</td>
<td>228.378</td>
<td>25.978</td>
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<td>0.254</td>
<td>0.027</td>
<td>0.105</td>
<td>28.547</td>
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<tr>
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<td>0.127</td>
<td>0.053</td>
<td>0.420</td>
<td>3.568</td>
<td>0.406</td>
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<tr>
<td>0.070</td>
<td>3</td>
<td>0.073</td>
<td>0.093</td>
<td>1.287</td>
<td>0.666</td>
<td>0.076</td>
</tr>
<tr>
<td>0.100</td>
<td>2</td>
<td>0.051</td>
<td>0.133</td>
<td>2.627</td>
<td>0.228</td>
<td>0.026</td>
</tr>
</tbody>
</table>
Overheating of laser with short wide unstable cavity

The surface loss $\beta$ limits also the size and power of the laser with short wide unstable cavity [7].

For the example with Yb, at $\beta = 0.05$, the robust operation is possible only in the pulsed regime. For the efficient cw operation, $\beta$ should be reduced to 0.01; then the magnification coefficient can be reduced from 1.2 to 1.05, and the size of the laser can be increased from 2 mm to 8 mm.
Maximal power of a disc laser is determined by ratio $\omega_s/\omega_p$, saturation parameter $Q$, overheating parameter $R$, and the surface loss $\beta$. Power $P_D = \frac{R^2\eta_o}{Q\beta^3}$ is key parameter in the choice of material for a high power disc laser.

The maximal power $P_{s,\text{max}} = \frac{27}{64e^2} P_D \approx 0.06 P_D$ corresponds to the size $L = \frac{9}{8e} R/Q \approx 0.4 R/Q$ and efficiency $\sim 10\%$.

The efficient operation is possible at power smaller than $10^{-3} P_D$.

The scaling up the size and power of a disc laser requires moderate scaling down the surface loss $\beta$. 
LIMITS OF THE MODEL and EXTENSIONS

Our estimates apply also to an amplifier, then $g$ becomes gain per single element. Similar estimate takes place for the pulsed lasers.

Additional limitations may be caused by the problems of efficient absorption of pump in the optically-thin medium, dissipation of the Amplified Spontaneous Emission, and deformation of the active mirror due to the thermal loading.

The widely scalable architecture of powerful laser may include the active adjustment of phase of each of cheap and efficient partial lasers to some master oscillator which plays the role of clock operating at optical frequency. Minimal power of the reference signal required to adjust the phase of a partial laser should be estimated.


