

Storage of energy in disk-shaped laser materials

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Dmitrii Kouznetsov

Institute for Laser Science, University of Electro-Communications

1-5-1 Chofu-Gaoka, Chofu, Tokyo, 182-8585, Japan <http://www.ils.uec.ac.jp/~dima>

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Thin disk laser is analyzed, assuming that the heat is drained in the same direction, as the optical pulse withdrawing the stored energy. Amplified spontaneous emission, the background loss and overheating are taken into account with simple model. The scaling laws of the basic parameters are deduced. For the case of fixed repetition rate, the upper bound of thickness is obtained. Key parameters are suggested. The key energy parameter is promoted as criterion for evaluation of different laser materials for the high energy, high mean power disk lasers. The maximum energy per active element is estimated. For the scaling up the power and the energy withdrawn from a single active element, the background loss should scale down inversely proportional to the cube of the background loss. This scaling law gives the criterion whether the heat should be drained in the direction orthogonal to the beam that withdraws the energy.

I. INTRODUCTION

Nobody can tabulate numerically performance of a laser as a function of tens of variables, describing the size and shape of the pumped region, spectra of pump and that of signal, the spatio-temporal profile and so on. The qualitative analysis, that allows simple estimate of the basic parameters in the analytic form, is important complementary to the detailed simulation. Such estimates cannot be substituted by the detailed simulations. (Similarly, ultra-high resolution objective cannot substitute a low-NA lupa with long depth of field). The result of a qualitative analysis can be expressed with analytic estimate, or “scaling law”.

In this paper, *scaling law* means a *mathematical expression, that shows the general trend of variation of parameters of a laser, as it is designed for higher and higher power, or higher and higher energy.*

The scaling law appears, when only few physical effects

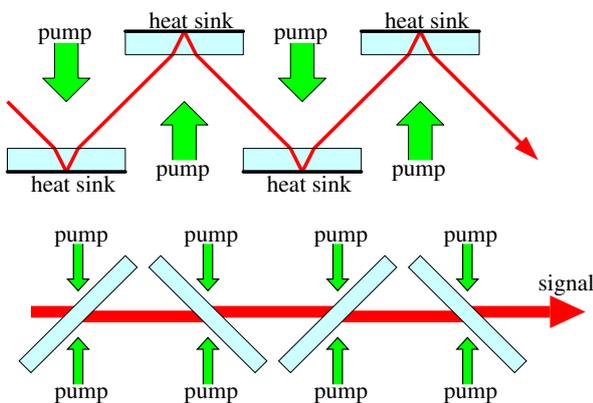


FIG. 1: Set of active mirrors and set of active slabs, crossed with signal that withdraws the energy.

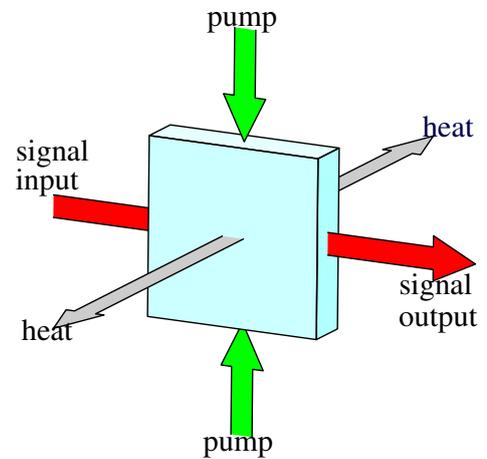


FIG. 2: Robust configuration of slab laser.

are dominant in the limit of the power or energy. For identification of dominant mechanisms in the scaling of energy, various effects can be estimated independently. In this paper, the limit due to the ASE, overheating and the background loss is considered in the simplest possible form; other effects are neglected.

Propagation of the amplified light across the thin slab of active medium is typical for the disk lasers, whenever the active mirror configuration [1–3] is used, or the light just crosses the set of pumped slabs (fig.1). In these cases, the amplification of the signal is weak compared to that of the Amplified Spontaneous Emission (ASE), which can have long path in the gain medium. The special efforts are required to suppress the ASE, and give all the advantages to the signal, propagating across the slab. In particular, the signal, going across the medium, should have small background loss.

For the continuous-wave disk lasers, roughly, at the

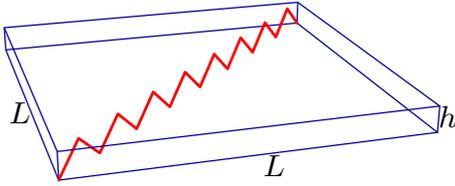


FIG. 3: Sizes of the active slab and the longest bouncing ray.

power scaling, the loss should be reduced inversely proportional to the cubic root of the desired power [4–6]. For power scaling at fixed value of the background loss, the heat should be drained in the direction orthogonal to the propagation of the laser beam (Fig.2); then other mechanisms (perhaps, deformation of wave front due to nonlinear refraction) may become dominant.

Various processes can contribute to the background loss, including absorption and scattering at unwanted dopants inside the bulk medium. The scattering at the surface seems to be most important among dissipation processes that contribute to the background loss. For the estimate of the maximal mean power, the background loss determines minimal gain, at which the energy still can be withdrawn from the excited medium; through ASE, this gain limits the size of the pumped region; then, ability to drain the heat limits the mean power.

In this paper, assuming the fixed repetition rate, I show, that similar relation keeps for pulsed operation; the background loss should be reduced inversely proportional to the cubic root of the desired energy. This case is important for the projects of nuclear fusion power plant with laser driving [7–9]. This may be considered as an argument in favor of configuration shown in Fig.2., where the signal light propagates in the active medium in the direction orthogonal to the direction of drain of heat.

II. EFFECTIVE PATH OF ASE

ASE is especially important at the stage of storage of the energy in the active medium, just before the signal pulse comes. Assume, that the ASE is efficiently absorbed (or even recycled) at the edges of the active medium. Then, its effect on the excitation can be taken into account with scaling of the effective lifetime τ that becomes $\tau \exp(-GS)$, where G is gain and S is some effective path of ASE in the gain medium. Following [4–6], assume, that the effective path S is of order of half of the maximal path of ASE in the medium. More optimistic estimate for S (quarter of the maximal path) is suggested in the comment [11], but the correction coefficient is not yet justified. Due to internal reflection(Fig.1), for the rectangular slab, the maximal path can be of order of $2L$; so, I take $S = L$ for the estimates.

The evolution of number N of excitations in the gain

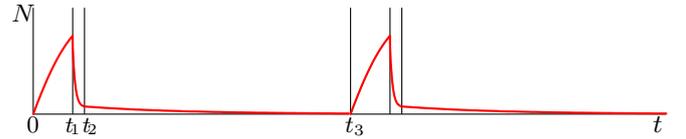


FIG. 4: Periodic evolution of number N of excitations.

medium can be approximated with equation

$$\frac{dN}{dt} = \frac{P}{\hbar\omega_p} - N \exp(-GL)/\tau, \quad (1)$$

where P is power of pump, absorbed in the gain medium and $\hbar\omega_p$ is energy of photon of pump. Assume that pump pulse acts during time t_1 , then, during the short time $t_2 - t_1$ the most of energy, stored in the gain medium is withdrawn with a by-passing pulse at the lasing frequency, and the slab is allowed to cool during time t_3 before the next pulse (fig.4). Gain G can be estimated as follows:

$$G = \frac{N\sigma}{L^2 h}. \quad (2)$$

where σ is effective emission cross-section at the lasing frequency. As in the case of cw operation, it is convenient to use dimension-less variable $u = GL$, that has sense of the transverse-trip gain. Combining equations (1) and (2), I get

$$\frac{du}{dt} = \frac{P\sigma A}{\hbar\omega_p L^2} - u \exp(u)/\tau, \quad (3)$$

where $A = L/h$ is aspect ratio of the slab of the active medium; $A \gg 1$. The steady-state solution $u = u_1$ of equation (3) can be expressed as follows:

$$u_1 \exp(u_1) = \frac{\sigma A \tau}{\hbar\omega_p L^2} P. \quad (4)$$

For analytical estimates of orders of magnitude, I assume that the transverse-trip of order of value u_1 is achieved after time of order of

$$t_1 = \frac{N_1 \hbar\omega_p}{P} = \tau \exp(-u_1). \quad (5)$$

There is no reason to keep the medium pumped much longer than time t_1 , the additional increase of inversion of population is exponentially small.

III. LASER ACTION

Assume, that after time t_1 by (5), the strong pulse comes in order to withdraw the energy of excitation of the medium. During this pulse, the evolution of number N of excitations in the gain medium and that of number W of photons in the pulse can be described with system

$$\begin{cases} \frac{dN}{dt} = -gF \\ \frac{dW}{dt} = (g - \beta)F \end{cases}, \quad (6)$$

where β is background loss, $F = F(t)$ is flux of photons at the lasing frequency ($\hbar\omega_s F$ is power in pulse that withdraws the energy), and $g = Gh = u/A$ is the single-trip gain, which is assumed to be small, $g \ll 1$. This assumption is justified by the condition $A \gg 1$; at large values of g , u is huge, and the ASE becomes exponentially strong. As the withdrawal is quick, I neglect both the ASE and the pumping (even if it is still on until time t_2) during the laser action. Then

$$\frac{dW}{dN} = \frac{g - \beta}{-g} = -1 + \frac{\beta L^2}{\sigma N}. \quad (7)$$

Assume, at the beginning of the pulse (time t_1), the number of excitations $N = N(t_1) = N_1$, and at the end of the pulse, $N = N(t_2) = N_2$. Let the number of photons in the pulse be W_1 before it passes through the gain medium, and W_2 after the amplification. The number of withdrawn photons can be expressed as follows:

$$w = W_2 - W_1 = \left(-N + \frac{\beta L^2}{\sigma} \ln N \right) \Big|_{N=N_1}^{N=N_2}. \quad (8)$$

The number of excitations is related with gain;

$$N = \frac{L^2}{\sigma A} u = \frac{L^2}{\sigma} g. \quad (9)$$

Assume the maximal excitation at the arrival of pulse:

$$N_1 = \frac{L^2}{\sigma A} u_1. \quad (10)$$

This is optimistic estimate, because the maximal number of excitations is achieved only asymptotically, while the most of pump energy is dissipated with ASE.

Let the pulse ends, when the round-trip gain g becomes equal to loss β . Then

$$N_2 = \frac{L^2}{\sigma} \beta. \quad (11)$$

Substituting (10) and (11) into (8) gives

$$w = \frac{uL^2}{\sigma A} \left(1 - \frac{\beta A}{u_1} + \frac{\beta A}{u_1} \ln \frac{\beta A}{u_1} \right). \quad (12)$$

Expressing L^2 from (4), I estimate the withdrawn energy:

$$E = \hbar\omega_s w = \frac{\omega_s}{\omega_p} P \tau \exp(-u_1) \eta \left(\frac{\beta A}{u_1} \right), \quad (13)$$

where η is elementary function,

$$\eta(z) = 1 - z \ln \frac{e}{z}, \quad (14)$$

shown in Figure 5. It has sense of quantum efficiency; its argument

$$z = \frac{\beta A}{u_1} = \frac{\beta L}{u_1 h} \quad (15)$$

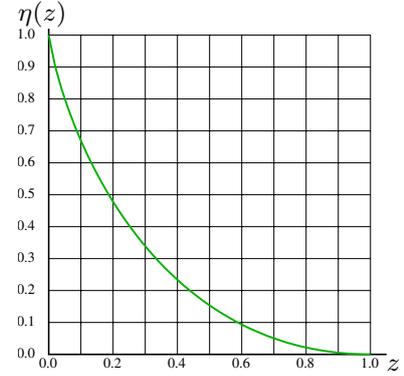


FIG. 5: Quantum efficiency $\eta(z)$ versus z by equation (14).

determines the self-similarity of pulsed disk lasers. For the efficient operation, parameter z should be small, $z \ll 1$. For example, at $z > 0.01$, the quantum efficiency cannot exceed 95%; and at $z > 0.1$, the quantum efficiency cannot exceed 67%.

At the self-similar scaling of energy, the product of the background loss β to the aspect ratio $A = L/h$ should remain small constant; this should be interpreted as *law of scaling of energy of disk lasers*.

IV. HEAT REMOVAL

The time t_3 of cooling after a pulse is determined by the repetition rate. For example, repetition rate of several Hertz is required for the application in the laser nuclear fusion electric plant [7–9]. There is important question, how much energy can be withdrawn from a single active element at given repetition rate, without overheating. Such a maximal energy may be limiting factor for the application of the disk lasers, while the heat sink is realized in the same direction, as the propagation of the signal pulse. In this section, I estimate the maximal energy that can be withdrawn at given repetition rate $1/t_3$.

The maximal mean power of pump, that can be delivered to the active slab, is proportional to the area and inverse proportional to its thickness. The coefficient R of proportionality is the thermal loading parameter [4–6]:

$$\frac{t_1}{t_3} P = R \frac{L^2}{h}. \quad (16)$$

Two mechanisms may contribute to this limitation: the thermal shock and the overheating [12, 13]. At strong pumping, the slab may crack; and even if it does not, the laser action becomes non-efficient, if the temperature becomes comparable with the quantum defect. There is no established value for the coefficient R ; the estimates may vary for orders of magnitude, but this quantity is limited for any given laser material.

Using estimate $t_1 = \tau \exp(-u_1)$ and definition of $A =$

L/h , the equation (16) can be re-written as follows:

$$P = \frac{RA t_3}{\tau} L \exp(u_1). \quad (17)$$

Equations (17), (13) and (4) determine the basic properties of the disk lasers, at the scaling up the withdrawn power.

V. SCALING LAWS

The deduction can be drastically simplified, using the dimension-less variables. Let

$$E_d = \frac{\omega_s R^2 \sigma t_3^2}{\omega_p \hbar \omega_p} \quad (18)$$

$$P_d = \frac{R^2 \sigma t_3^2}{\hbar \omega_p \tau} \quad (19)$$

$$L_d = \frac{R \sigma t_3}{\hbar \omega_p} \quad (20)$$

Use of variables

$$\mathcal{E} = E/E_d, \quad p = P/P_d, \quad \ell = L/L_d \quad (21)$$

allows to express all the parameters in terms of β , z and u_1 . Equations (17), (13) and (4) become

$$p = A^3 \exp(u_1)/u_1 \quad (22)$$

$$\mathcal{E} = A^3 \eta(z)/u_1 \quad (23)$$

$$\ell = A^2/u_1 \quad (24)$$

Using (15) and $h = L/A$ gives

$$\frac{P}{P_d} = p = \left(\frac{z}{\beta}\right)^3 u_1^2 \exp(u_1) \quad (25)$$

$$\frac{E}{E_d} = \mathcal{E} = \left(\frac{z}{\beta}\right)^3 u_1^2 \eta(z) \quad (26)$$

$$\frac{L}{L_d} = \ell = \left(\frac{z}{\beta}\right)^2 u_1 \quad (27)$$

$$\frac{h}{L_d} = \frac{z}{\beta} \quad (28)$$

From the equation (28) it follows, that no disk laser can operate at thickness $h \geq L_d/\beta$, because $z < 1$;

$$h_k = \frac{L_d}{\beta} = \frac{R \sigma t_3}{\hbar \omega_p \beta} \quad (29)$$

is key parameter for the thickness of the pulsed disk laser. In order to operate with repetition rate $1/t_3$ the disk laser should be thinner than h_k . Similarly,

$$E_k = \frac{E_d}{\beta^3} = \frac{\omega_s R^2 \sigma t_3^2}{\omega_p \hbar \omega_p \beta^3} \quad (30)$$

$$P_k = \frac{E_d}{\beta^3} = \frac{\omega_s R^2 \sigma t_3^2}{\omega_p \hbar \omega_p \tau \beta^3} \quad (31)$$

$$L_k = \frac{E_d}{\beta^2} = \frac{R^2 \sigma t_3}{\hbar \omega_p \tau \beta^3} \quad (32)$$

are key parameters for the energy of output pulses, mean power, pump power and the size. It is notable that within this model, the maximal mean power E_k/t_3 increases with decrease of the repetition rate $1/t_3$; however, this dependence will be broken as soon as the increase of temperature per single pulse becomes sufficient to cause the overheating of the device.

The exceed of energy above the key parameter implies loss of efficiency and/or exponential growth of the required pump power. For the efficient operation, parameters z and u_1 , should remain small. Such a scaling law is an analogy of the scaling law by [15], that considered the optimization of maximal energy per active element at fixed concentration of active centers.

The similar law of power scaling for cw operation is considered in [4–6]; for that case, for the efficient operation, the background loss should scale down inversely proportional to the cubic root of the desired output power. The application of the undoped cap could allow to increase the mean power with a factor or order of $\log(\beta)^2$, as in the case of cw operation [5], giving an additional order of magnitude in the energy scaling; however, such a prediction implies, that addition of the thick undoped cap does not increase the background loss β .

I should greatly appreciate the confirmation of the scaling laws both for pulsed and the cw disk lasers with detailed numerical simulations and/or direct experiments. Especially important is careful measurement of the thermal loading parameter R and the background loss β ; the estimates of the maximal power and maximal energy are proportional to R^2/β^3 .

VI. ROBUST CONFIGURATION

The upper bound for the energy above reverts to the specific case of given repetition rate and given background loss and given geometry (fig.1) of withdrawal of energy and drain of heat. This upper bound is not general limit for solid-state lasers. The energy can be withdrawn in the direction, orthogonal to the direction of heat sink. Then, the efficiency is not so sensitive to the background loss β .

Parameter u determines, how fast the energy should be delivered to the gain medium. The pump power required grows exponentially with u_1 ; so, values of u_1 of order of unity or even smaller look reasonable. However, if the power available exceeds ratio E/τ for orders of magnitude, larger values of $u_1 > 1$ may be used too; but in this case, it may have sense to distribute the available power among several lasers in such a way that each one works at small values parameters u_1 and z .

Assuming fixed u_1 , from equation (2) and (16) the required size L can be estimated as follows:

$$L = \left(\frac{E^2 \sigma \hbar \omega_p}{(\hbar \omega_s)^2 t_3 R u_1} \right)^{1/3}. \quad (33)$$

at thickness of order of

$$h = L^2 \frac{R t_3 \hbar \omega_s}{E \hbar \omega_p}. \quad (34)$$

At long interval between pulses

$$t_3 \geq \frac{E \omega_s}{R \omega_p L}, \quad (35)$$

there is no need to make the medium thin-disk shaped, it could be a cube (or cylinder) as well; the medium has enough time to cool and the estimates above are not valid. In such a way, the estimates (33) and (34) have sense for the configuration shown in Figure 2 at high repetition rate or high energy of pulses, i.e., at high mean power. Even in this case, the ASE, overheating and background loss do not set general limit of the energy that can be withdrawn from a single active element. This may be important for application of the ceramic materials that allow to manufacture wide pieces of active media.

VII. CONCLUSIONS

The scaling of energy of a slab laser is considered under assumption that the drain of heat occurs in the same direction as propagation of the amplified light. The exponential growth of ASE is described with effective lifetime $\tau \exp(-u)$, where u is transverse-trip gain. The background loss β and thermal loading parameter R are used

to characterize the laser material. Repetition rate $1/t_3$ is assumed to be fixed parameter, that follows from the intended application of the pulsed disk laser. For the energy scaling up, the background loss should be scaled down inversely proportional to the cubic root of the desirable energy. Over vice, the pump power required scales up as exponential of the square root of the output energy.

The scaling laws can be expressed in terms of the key parameters (26),(25),(27),(28). For the efficient operation, the energy, power, size, and thickness should remain small, compared to the key parameters.

The approach suggested seems to be applicable to any laser material, for which the thermal loading R and cross-section σ can be estimated. No active mirror (Fig.1) can operate at thickness larger than h_k . The key parameter E_k should be used at the evaluation of laser materials for the energy scaling of disk lasers with fixed repetition rate. Such evaluation is important for the design of the driver for the nuclear fusion power plant. If the required scaling down of the background loss is not possible, then another architecture of the laser should be considered, where the light, withdrawing the energy, passes the slab in the direction, orthogonal to the direction of drain of heat [14], as it is shown in Fig.2.

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- [1] J. A. Abate, L. Lund, D. Brown, S. Jacobs, S. Reformat, J. Kelly, M. Gavin, J. Waldbillig, and O. Lewis. Active mirror: a large-aperture medium-repetition rate Nd:glass amplifier Appl. Opt. **20**, 351-361 (1981)
 - [2] K. Ueda and N. Uehara, "Laser-diode-pumped solid state lasers for gravitational wave antenna", in Proc. SPIE **1837**, 336-345 (1993)
 - [3] A. Giesen, H. Hügel, A. Voss, K. Wittig, U. Brauch, and H. Opower, "Scalable concept for diode-pumped high-power solid-state lasers", Appl. Phys. **B 58** (5), 365-372 (1994)
 - [4] D. Kouznetsov, J.-F. Bisson, J. Dong, and K. Ueda, "Surface loss limit the power scaling of a thin disk laser", J. Opt. Soc. Am. B, **23** 1074-1082 (2006)
 - [5] D. Kouznetsov, J.-F. Bisson. Role of undoped cap in the scaling of thin disk lasers, J. Opt. Soc. Am. B **25**, 338-345 (2008)
 - [6] D. Kouznetsov, J.-F. Bisson, K. Ueda. Scaling laws of disk lasers. Optical materials, in press. (2008) doi:10.1016/j.optmat.2008.03.017
 - [7] T. Kawashima et al. The HALNA project: Diode-pumped solid-state laser for inertial fusion energy. J. Phys. IV France **133**, 615-620 (2006)
 - [8] K.Naito, M.Yamanaka, M.Nakatsuka, T.Kanabe, K.Mima, C.Yamanaka, S.Nakai. Conceptual design of a laser diode pumped solid state laser system for the laser fusion reactor driver. Jpn.J.Appl.Phys. **31**,259-273(1992)
 - [9] C. D. Orth, S. A. Payne, and W. F. Krupke. A diode pumped solid state laser driver for inertial fusion energy. Nuclear Fusion **36**, 75-116, (1996)
 - [10] T.Y.Fan. Optimizing the efficiency and stored energy in quasi-three-level lasers. IEEE J.of QE, **28**, 2692-2697 (1992)
 - [11] R. Paschotta, J. Speiser, and A. Giesen. "Comment on "Surface loss limit of the power scaling of a thin-disk laser", J. Opt. Soc. Am. B **24**, 2658-2658 (2007)
 - [12] B. Henderson and R. H. Bartram, *Crystal-field engineering of solid-state materials*, (Cambridge University Press, 2000).
 - [13] W.F.Krupke, M.D. Shinn, J.E. Marion, J.A. Caird, and S.E. Stokowski. Spectroscopic, optical, and thermomechanical properties of neodymium- and chromium-doped gadolinium scandium gallium garnet. J. Opt. Soc. Am. B **3** 102-114. (1986)
 - [14] T.Kawashima, T.Kanabe, H.Matsui, T.Eguchi, M.Yamanaka, Y.Kato, M.Nakatsuka, Y.Izawa, S.Nakai, T.Kanzaki and H.Kan. Design and Performance of a Diode-Pumped Nd:Silica-Phosphate Glass Zig-Zag Slab Laser Amplifier for Inertial Fusion Energy. Jpn. J. Appl. Phys. **40**, 6415-6425(2001)
 - [15] J.Degnan. Theory of the Optimally Coupled Q-Switched Laser. IEEE J. of QE, **25**, 214-220 (1989)