Boundary behavior of modes of Dirichlet Laplacian

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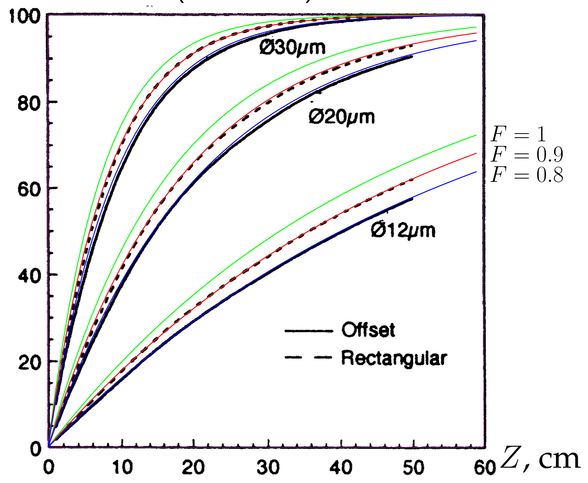
The problem of efficient use of pump in double-clad fibers refers to the modes of Dirichlet Laplacian. Few theorems about the boundary behavior of such modes are suggested. They indicate the especially good properties of the spiral-shaped cladding. These properties of the boundary behavior of modes could be used for the whispering-gallery semiconductor lasers or even for cooling of trapped atoms.

Introduction

The length of powerful fiber lasers is limited by the processes of Raman conversion and stimulated scattering.
The cost of pump increase with the requirement of brightness.
One of solution is the transverse delivery of pump [1,2].
For justification of the advantages of the transversal delivery of pump, it is important to show limits of the conventional delivery.
The spiral-shaped cladding seems to realize this limit (at given area of cladding and size of core).

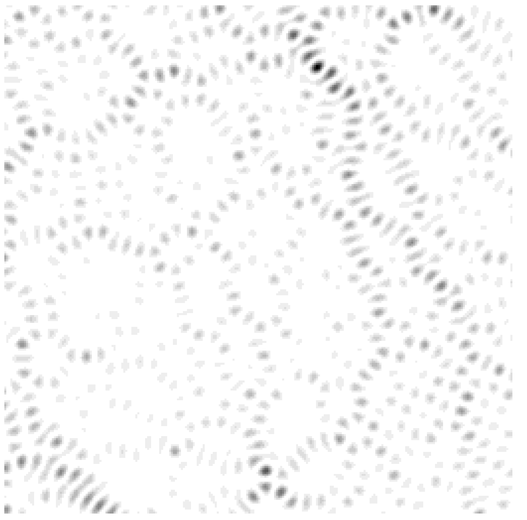
The chaotic fibers allows the exponential approximation of the efficiency

$$\eta = 1 - \exp\left(-\frac{\pi r^2}{S}\alpha Z\right)$$

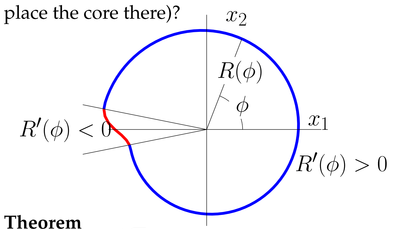


Example from [3], ray tracing (black) [4] and the exponential fit with different F (colored).

Scars are typical for modes of Dirichlet Laplacian [5].



How to force all the modes to have scars in some region (and place the core there)?



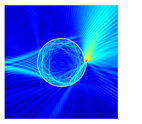
Theorem

$$\int_{-\pi}^{\pi} \left(\Psi'\right)^2 R(\phi)\, R'(\phi)\, \mathrm{d}\phi$$

All modes "visit" chunk where $R' < 0$ (see [6,7]).

Other applicaitons:

Spiral-shaped whispering-gallery laser by [8].



spiral-shaped acoustical drum would allow efficient excitation of all modes with single kick

Conclusions.

Good coupling of pump at the spiral-shaped cladding with core in vicinity of the chunk.

Boundary properties of modes couple all of them to the chunk providing the unidirectional output from a whispering-gallery mode lasers.

References

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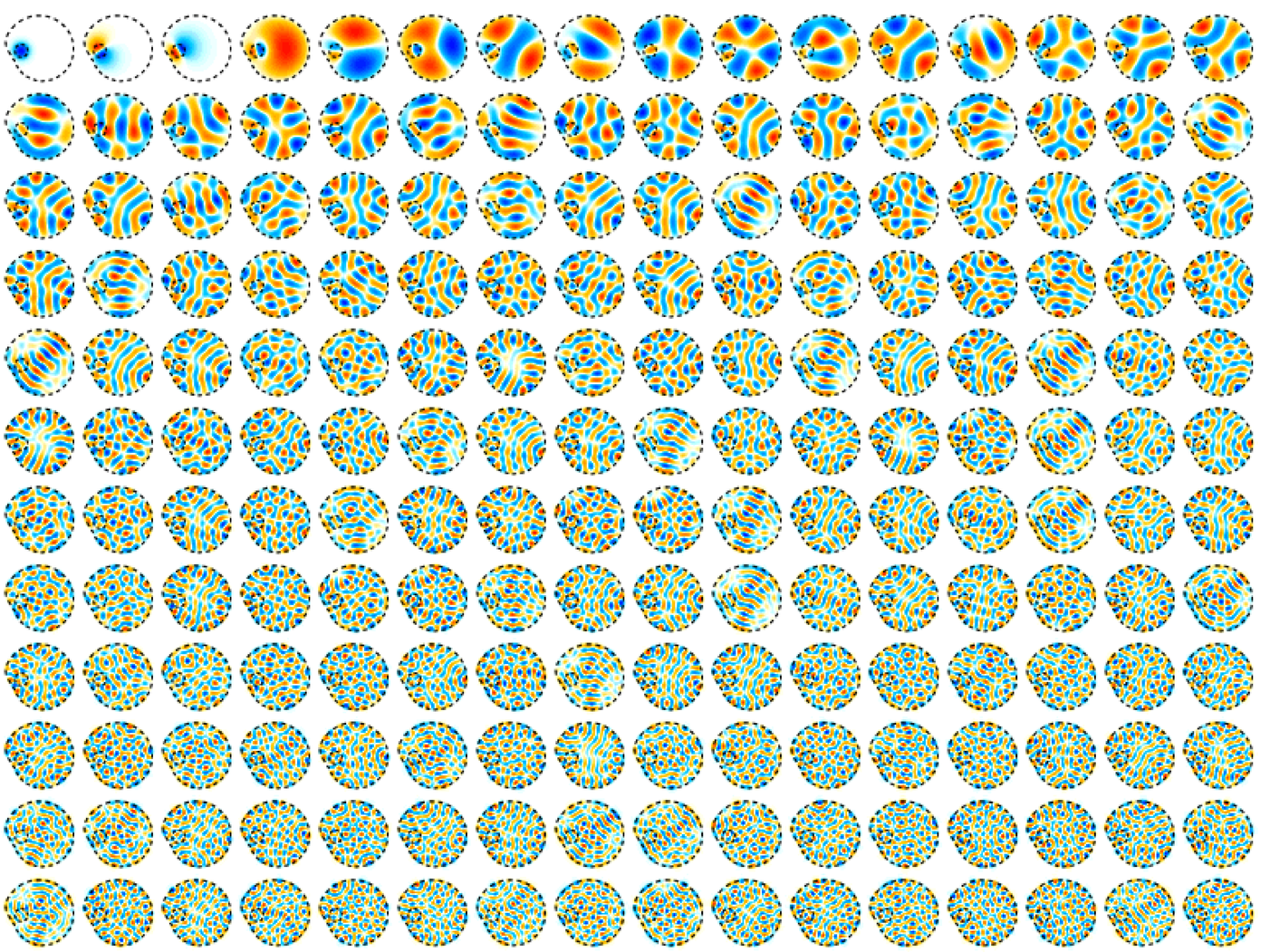
Pump modes of a spiral-shaped double-clad fiber [6]