Scaling laws of disk lasers

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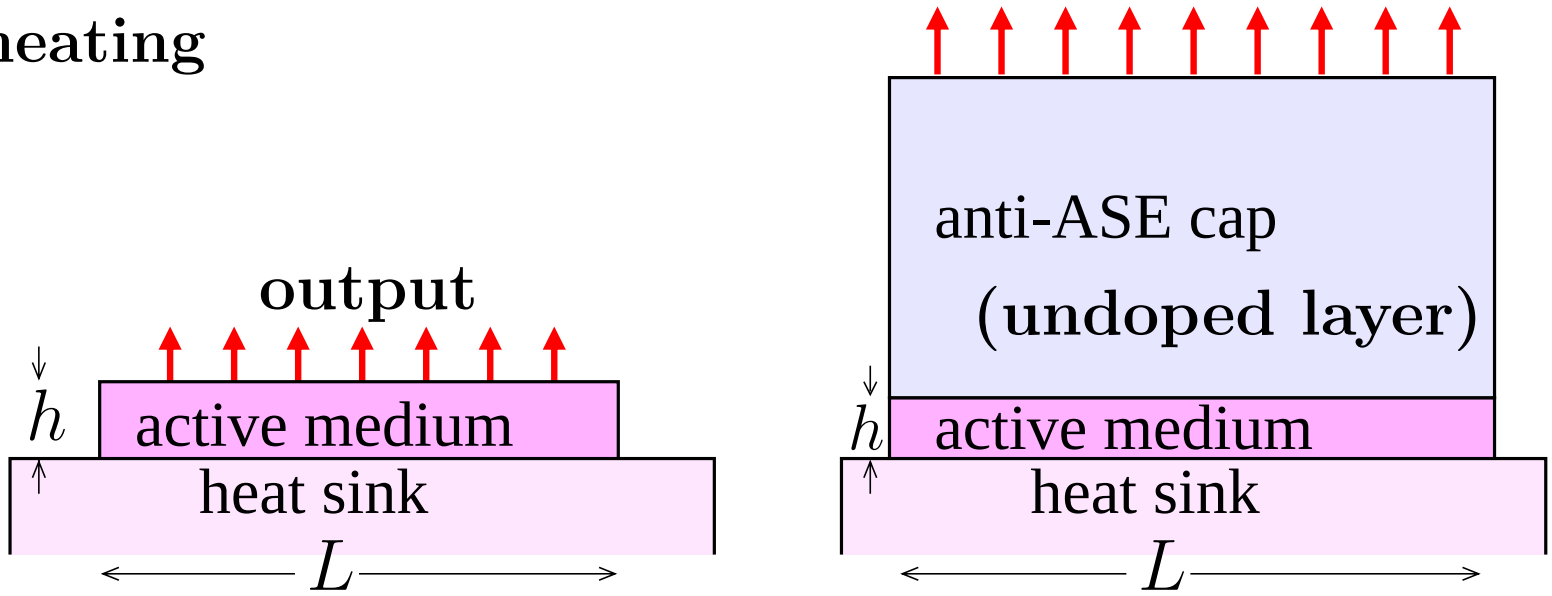
Limit of power from

Amplified Spontaneous Emission (ASE),
round-trip loss,
and overheating

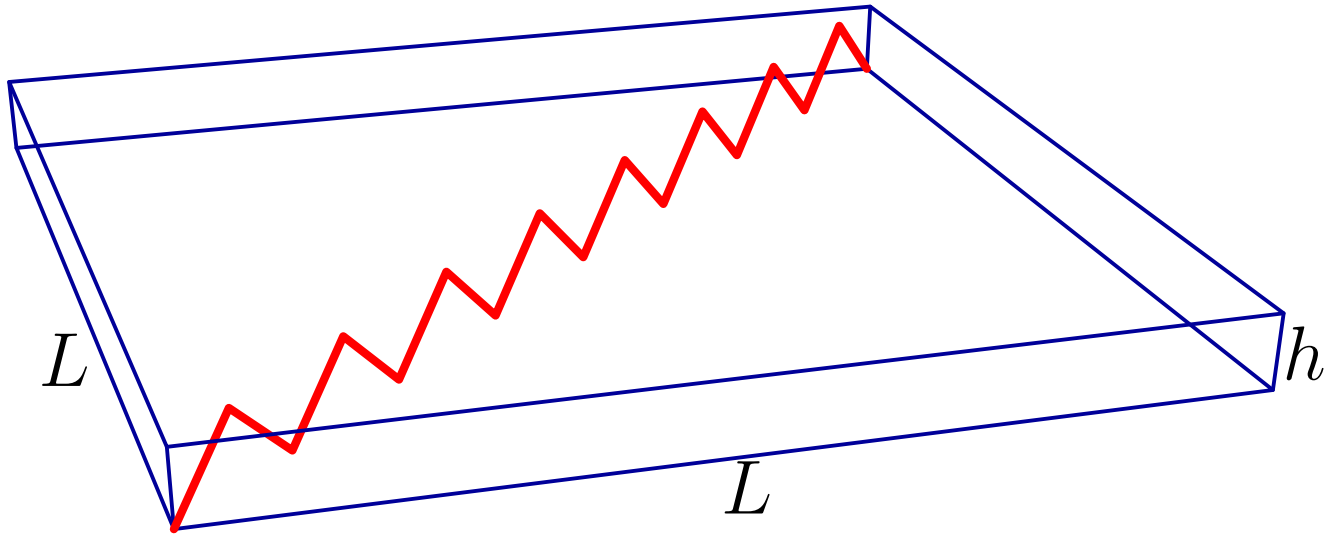


D.Kouznetsov, J.-F.Bisson, J.Dong, K.Ueda.
Surface loss limit of the power scaling of a thin-disk laser. JOSA B 23, p.1074-1082 (2006)

D.Kouznetsov, J.-F.Bisson.
Role of anti-ASE cap in the scaling of thin disk lasers. JOSA B, under consideration.

Amplified Spontaneous Emission

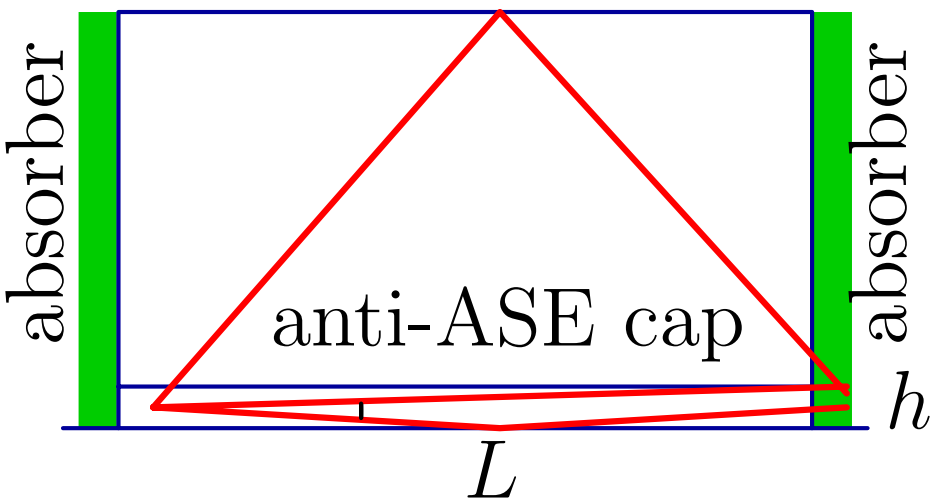
Extremal case of a ray of length $2L$ bouncing along the diagonal in a block of active medium $L \times L \times h$ with total internal reflection



Strong assumption:
Effective path of ASE is L .

Effective lifetime, no cap: $\tau = \tau_o \exp(-GL)$

At the anti-ASE cap
the only portion proportional to the angle h/L remains within the active medium before to reach the absorber at the edge.



with cap: $$\frac{1}{\tau} = \frac{1}{\tau_o} + \frac{h}{L}\frac{\exp(GL)}{\tau_o}$$

Transverse-trip gain, Round-trip gain and the round-trip loss

http://en.wikipedia.org/wiki/Round-trip_loss

For the accurate consideration with geometric optics, one should define distribution of gain $G(x, y, z)$ and the path gain along each path $X(a)$, $Y(a)$, $Z(a)$ is determined with the integral

$$\int G(x(a), y(a), z(a))\ \mathrm{d}a$$

For qualitative estimates, the round-trip gain $g = 2Gh$

After to pass the gain medium, intensity I becomes $I \exp(g)$

Assume, the only small part of the energy of the light in the cavity is outputted at each round-trip. The intensity I after the output coupler becomes $(1 - \theta)I$

Assume, the background loss β at each round-trip reduces the intensity with factor $(1 - \beta)$
For cw operation, the round-trip gain g compensates both, the output coupling θ and the background loss β .

At $\beta \ll 1$, $\theta \ll 1$,

$$g = \beta + \theta$$

while the loss is limited bounded below by technical limitations, the gain is **determined** by the output coupling parameter θ.

For optimization at given β, g is good as independent parameter

Thermal loading and overheating

Thermal shock parameter $R_\mathrm{T} = \frac{k\sigma_\mathrm{T}(1-\nu)}{\alpha E}$

k is thermal conductivity,

σ_T is maximal tension the material can resist,

α is the thermal expansion coefficient

E is the young's modulus, and

ν is the Poisson ratio.

$$P_\mathrm{p,max} = 3\frac{R_\mathrm{T}}{q}\frac{L^2}{h}$$

where $q \approx 1 - \frac{\omega_\mathrm{p}}{\omega_\mathrm{s}}$

The maximal raise ΔT of temperature allows pump power

$$P_\mathrm{p,max} = 2\frac{k\Delta T}{q}\frac{L^2}{h}$$

Combination gives:

$$P_\mathrm{p,max} = R\frac{L^2}{h}$$

where

$$R = \min\begin{cases} 3R_\mathrm{T}/q \\ 2k\Delta T/q \end{cases}$$

is thermal loading parameter

W.F.Krupke; M.D.Shinn, J.E.Marion, J.A.Caird, S.E.Stokowski.
Spectroscopic, optical, and thermomechanical properties of neodymium- and chromium-doped gadolinium scandium gallium garnet. JOSAB**3**, 102-114 (1986)

http://en.wikipedia.org/wiki/Thermal_loading

Output power $P_\mathrm{s} = \eta_\mathrm{o}\left(1 - \frac{g}{\beta}\right)(P_\mathrm{p} - P_\mathrm{th})$

$$\eta_\mathrm{o} = \frac{\omega_\mathrm{s}}{\omega_\mathrm{p}}$$

Threshold power $P_\mathrm{th} = \frac{\hbar\omega_\mathrm{p}}{\tau}\frac{gL^2}{2\sigma}$

B.Henderson and R.H.Bartram, *Crystal-field engineering of solid-state materials*, (Cambridge University Press, 2000).

Define the saturation intensity $Q = \frac{\hbar\omega_\mathrm{p}}{2\tau_\mathrm{o}\sigma}$

Then

$$P_\mathrm{s} = \eta_\mathrm{o}\left(1 - \frac{g}{\beta}\right)\left(P_\mathrm{p} - QgL^2\frac{\tau_\mathrm{o}}{\tau}\right)$$

This expression can be maximized under condition $P_\mathrm{p} \leq RL^2/h$

for $\frac{\tau_\mathrm{o}}{\tau} = \exp\left(\frac{Lg}{2h}\right)$ (no cap)

and $\frac{\tau_\mathrm{o}}{\tau} = 1 + \frac{h}{L}\exp\left(\frac{Lg}{2h}\right)$ (with cap)

How many parameters do we have to play with?

Parameters for optimization: $g = 2Gh$, $u = GL$

Condition of cooling $\frac{L^2}{h} = \frac{P_\mathrm{p}}{R}$ $\qquad L = \frac{gP_\mathrm{p}}{2uR}$

Confition for gain: $\frac{L}{h} = \frac{2u}{g}$ $\qquad h = \frac{g^2 P_\mathrm{p}}{4u^2 R}$

$$P_\mathrm{s} = \eta_\mathrm{o}\left(1 - \frac{\beta}{g}\right)\left(P_\mathrm{p} - \frac{P_\mathrm{p}^2 Q}{R^2}\frac{g^3 \mathrm{e}^u}{4u^2}\right) \qquad \text{(no cap)}$$

$$P_\mathrm{s} = \eta_\mathrm{o}\left(1 - \frac{\beta}{g}\right)\left(P_\mathrm{p} - \frac{P_\mathrm{p}^2 Q}{R^2}\frac{g^3}{4u^2}\left(1 + \frac{g}{2u}\mathrm{e}^u\right)\right) \qquad \text{(with cap)}$$

$p = P_\mathrm{p}/P_\mathrm{d}$ $\qquad s = \frac{\omega_\mathrm{p}}{\omega_\mathrm{s}} P_\mathrm{s}/P_\mathrm{d}$ $\qquad P_\mathrm{d} = R^2/Q$

Key parameter: $P_\mathrm{k} = \eta_\mathrm{o}\frac{R^2}{Q\,\beta^3} = \frac{\omega_\mathrm{s}}{\omega_\mathrm{p}}\frac{R^2}{Q\,\beta^3}$

Case without cap, optimization at given P

$$P_\mathrm{s} = \eta_\mathrm{o}\left(1-\frac{\beta}{g}\right)\left(P_\mathrm{p} - \frac{P_\mathrm{p}^2 Q}{R^2}\frac{g^3 \mathrm{e}^u}{4u^2}\right)$$

$$s = \left(1-\frac{\beta}{g}\right)\left(p - p^2\frac{g^3\mathrm{e}^u}{4u^2\beta^3}\right)$$

maximum at $u = 2$

$$s = \left(1-\frac{\beta}{g}\right)\left(p - p^2\frac{g^3\mathrm{e}^2}{16\beta^3}\right)$$

maximization of s with respect to g gives

$$\frac{16/\mathrm{e}^2}{3-2\beta/g}\left(\frac{\beta}{g}\right)^4 = \beta^3 p$$

Then, optimal sizes: $L = \dfrac{gP_\mathrm{p}}{2uR}$, $h = \dfrac{g^2 P_\mathrm{p}}{4u^2 R}$

Case without cap, maximal power (no care about efficiency)

The maximum s corresponds to $\quad u = 2 \quad , \quad g = \frac{4}{3}\beta$

$$L = L_{\max} = \frac{r_{\mathrm{o}}}{8\mathrm{e}^2}\left(\frac{3}{\beta}\right)^2 \quad , \quad h = h_{\max} = \frac{r_{\mathrm{o}}}{8\mathrm{e}^2}\frac{3}{\beta}$$

$$p = p_{\max} = \frac{1}{8\mathrm{e}^2}\left(\frac{3}{\beta}\right)^3 \quad ; \quad s = s_{\max} = \frac{1}{64\mathrm{e}^2}\left(\frac{3}{\beta}\right)^3$$

Recovery of dimensional variables:

$$P_{\mathrm{p}} = P_{\mathrm{d}}\ p$$

$$P_{\mathrm{s}} = P_{\mathrm{d}}\ \tfrac{\omega_{\mathrm{s}}}{\omega_{\mathrm{p}}}\ s$$

$$P_{\mathrm{d}} = \frac{R^2}{Q} \quad , \quad r_{\mathrm{o}} = \frac{R}{Q}$$

$$P_{\mathrm{k}} = \frac{\omega_{\mathrm{s}}}{\omega_{\mathrm{p}}}\frac{R^2}{Q\beta^3} = \eta_{\mathrm{o}} P_{\mathrm{d}}/\beta^3$$

Optimisation with cap

$$P_\mathrm{s} = \eta_\mathrm{o}\left(1-\frac{\beta}{g}\right)\left(P_\mathrm{p} - \frac{P_\mathrm{p}^2 Q}{R^2}\frac{g^3}{4u^2}\left(1+\frac{g}{2u}\mathrm{e}^u\right)\right)$$

$$s = \left(1-\frac{\beta}{g}\right)\left(p - \frac{p^2 g^3}{4u^2}\left(1+\frac{g}{2u}\mathrm{e}^u\right)\right)$$

Set the derivatives with respect to u and g to zero;

$$\frac{4u\mathrm{e}^{-u}}{u-3} = g \quad , \quad \frac{2g^4 p(2\mathrm{e}^u g + 3u)}{3\mathrm{e}^u g^4 p + 4g^3 pu + 8u^3} = \beta \quad .$$

$$\frac{(u-3)^5 \mathrm{e}^{4u}\beta}{32u^2\left(6u-2-\beta u\mathrm{e}^u(u-3)\right)} = p$$

We do not need to solve any transcendental equation in order to plot optimal u versus p.

Maximal power for a disk with cap:

$$s = \left(1 - \frac{\beta}{g}\right)\left(p - \frac{p^2 g^3}{4u^2}\left(1 + \frac{g}{2u}\mathrm{e}^u\right)\right)$$

Maximization with respect to p:

$$p = \frac{4u^2}{g^3(\mathrm{e}^u g + 2u)} \quad , \qquad s = \frac{2(g-\beta)u^3}{g^4(\mathrm{e}^u g + 2u)}$$

Maximization with respect to u and g:

$$\frac{4\mathrm{e}^{-u}u}{u-3} = g \quad , \qquad \frac{2g(\mathrm{e}^u g + 3u)}{5\mathrm{e}^u g + 8u} = \beta \quad ;$$

The combination gives

$$\frac{2u\mathrm{e}^{-u}(3u-1)}{(2u-1)(u-3)} = \beta$$

Again, no need to solve a transcendental equation in order to plot optimal u versus β.

cap, asymptotic: $\mathrm{e}^u = \dfrac{3}{\beta}\dfrac{1-1/(3u)}{(1-1/(2u))(1-3/u)}$

$$u = \ln\tfrac{3}{\beta} + \ln\!\left(1-\tfrac{1}{3u}\right) - \ln\!\left(1-\tfrac{1}{2u}\right) - \ln\!\left(1-\tfrac{3}{u}\right) \ .$$

$\varepsilon = \frac{1}{\ln(3/\beta)}$

$$u \approx \tfrac{1}{\varepsilon} + \tfrac{19}{6}\varepsilon + \mathcal{O}(\varepsilon^2)$$

$$g \approx \tfrac{4}{3}\beta\left(1+\tfrac{1}{6}\varepsilon + \mathcal{O}(\varepsilon^2)\right)^{-1}$$

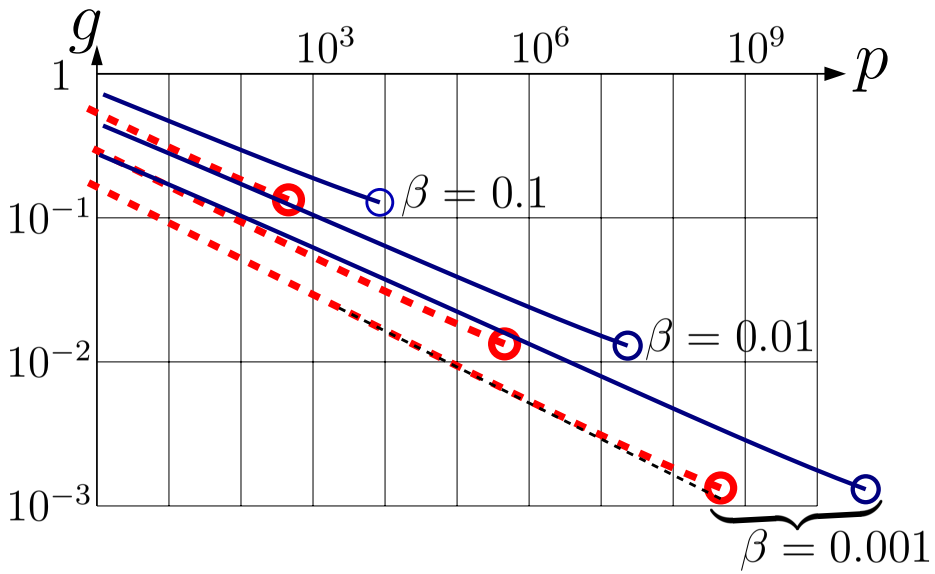
$$L \approx \tfrac{r_{\mathrm{o}}}{16}\left(\tfrac{3}{\beta\varepsilon}\right)^2\left(1+\tfrac{5}{3}\varepsilon + \mathcal{O}(\varepsilon^2)\right)^{-1}$$

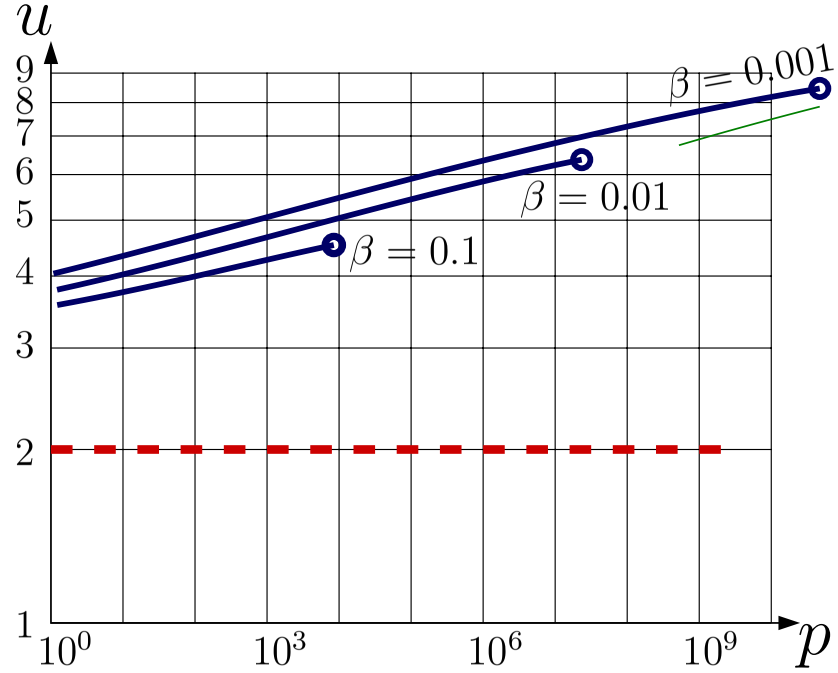
$$h \approx \tfrac{r_{\mathrm{o}}}{8}\left(\tfrac{3}{\beta}\right)\left(1+\tfrac{11}{3}\varepsilon + \mathcal{O}(\varepsilon^2)\right)^{-1}$$

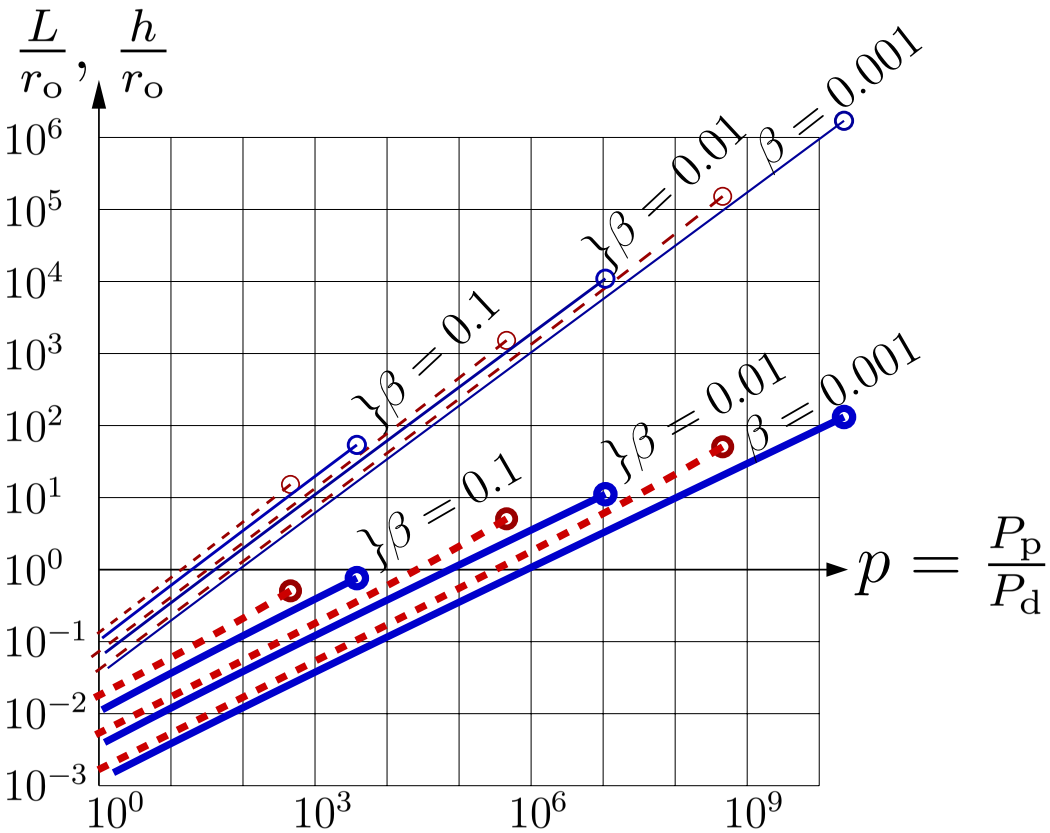
$$s \approx \left(\tfrac{3}{\beta}\right)^3 \tfrac{1}{256\varepsilon^2}\left(1+2\varepsilon + \mathcal{O}(\varepsilon^2)\right)^{-1}$$

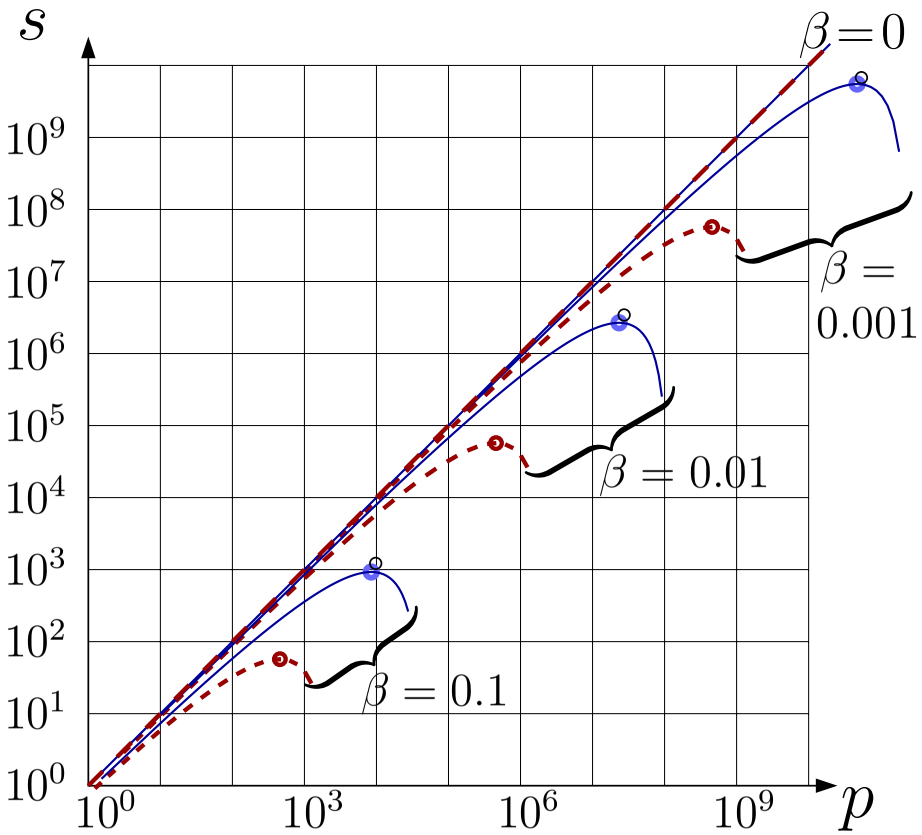
$$p \approx \left(\tfrac{3}{\beta}\right)^3 \tfrac{1}{32\varepsilon^2}\left(1+\tfrac{3}{2}\varepsilon + \mathcal{O}(\varepsilon^2)\right)^{-1}$$

Optimized parameters









Comparison with experimental data, recovery of β

Usually, β is not reported. Available:

θ is output coupling

$\eta_{\mathrm{o}} = \omega_{\mathrm{s}}/\omega_{\mathrm{p}}$

η_{a} is efficiency of absorption of pump estimated

η_{s} is slope efficiency achieved; $\eta_s = \eta_{\mathrm{o}}\eta_{\mathrm{a}}\left(1 - \dfrac{\theta}{g}\right)$

using $g = \beta + \theta$ get $\eta_s = \eta_{\mathrm{o}}\eta_{\mathrm{a}}\dfrac{\theta}{\beta + \theta}$

$$\beta = \theta\left(\frac{\eta_{\mathrm{o}}\eta_{\mathrm{a}}}{\eta_{\mathrm{s}}} - 1\right)$$

B. Henderson and R. H. Bartram, *Crystal-field engineering of solid-state materials*, (Cambridge University Press, 2000).

β and s from experimental data

Material \ unit	h mm	λ_p nm	λ_s nm	R $\frac{\text{Watt}}{\text{mm}}$	Q $\frac{\text{kW}}{\text{cm}^2}$	η_o %	θ %	η_a %	η_s %	P_s Watt	cite	P_d Watt	β %	s	row number
3%Yb:Lu_2O_3	0.25	976	1080	8	35	90.4	0.4	95	75	33	[1]	0.15	0.06	237	1
3%Yb:Lu_2O_3	0.25	976	1080	8	35	90.4	1.6	95	80	32	[1]	0.15	0.12	231	2
3%Yb:Lu_2O_3	0.25	976	1034	14	12	94.4	5.7	95	72	26	[1]	1.51	1.51	18	3
25%Yb:LSB	0.30	974	1040	3	36	94.0	1.0	99	48	40	[2]	0.03	1.38	1702	4
20%Yb:LSB	0.30	974	1040	3	36	94.0	1.0	96	38	36	[2]	0.03	0.94	1532	5
8%Yb:YAG	0.23	940	1030	6	5	91.2	2.2	95	56	480	[3]	0.68	1.37	775	6
9%Yb:YAG	0.22	940	1030	6	5	91.2	3.0	95	60	647	[3]	0.68	1.00	1044	7
10%Yb:YAG	0.20	940	1030	6	5	91.2	3.0	95	56	520	[4]	0.68	1.10	839	8
Yb:YAG	0.3	940	1030	6	5	91.2	3.0	95	70	5000	[5]	0.68	0.75	8001	9

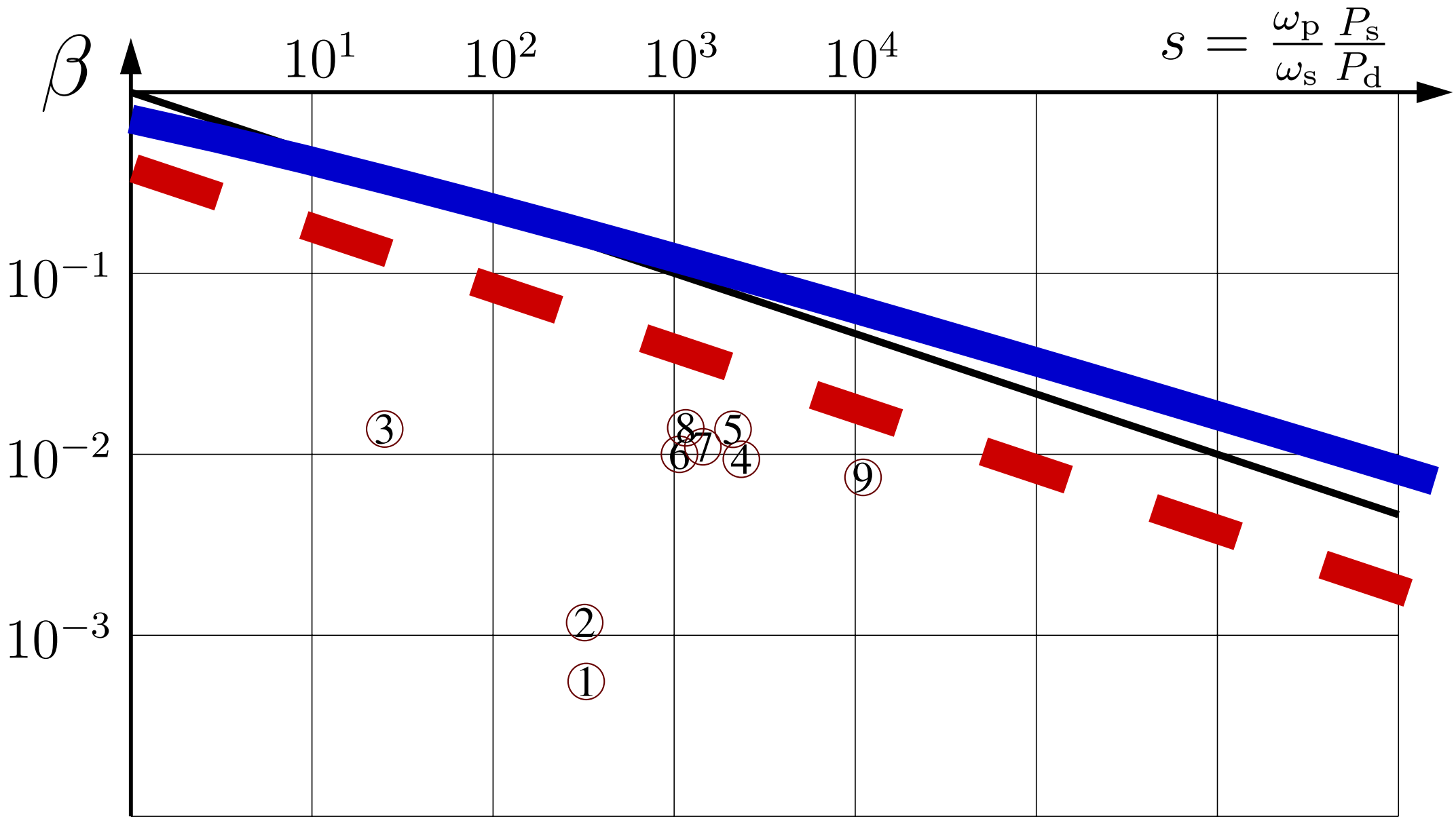
[1] R.Peters, C.Kränkel, K.Petermann, G. Hüber. Broadlu tunable high-power Yb:Lu_2O_3 thin disk laser with 80% slope efficiency. Optics Express, **15**(11), 78075-7082, 2007

[2] P.Kränkel, J.Johannsen, R.Peters, K.Peterman, G.Huber. Continuous wave high power laser operation and tunability of Yb:$LaSc_3(BO_3)_4$. Appl. Phys. **B 87**, 217-220, 2007

[3] C.Stewen, K.Contag, M.Larionov, A.Giesen, H.Huegel. IEEE J. Selec. Top. Quan. Electron. **6**(4), 450-657, 2000

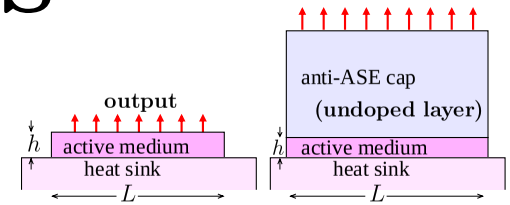
[4] M.Tsunekane, T.Taira. High-power operation of diode edge-pumped, composite all-ceramic Yb:$Y_3Al_5O_{12}$ microchip laser. Appl. Phys. Lett. **90**, 121101, 2007

[5] A.Giessen, L.Speiser, R.Peters, C.Kränkel, K.Peterman. Thin-disk lasers come of age. Photonics Spectra, May, p.52-58, 2007


β
10^1
10^2
10^3
10^4
$s = \frac{\omega_p}{\omega_s}\frac{P_s}{P_d}$
10^{-1}
10^{-2}
10^{-3}
3
8
5
6
7
4
9
2
1

Conclusions

Two configurations are compared.



The laser material is characterized with:

thermal loading R

saturation intensity Q

background loss β

Then, how wide and powerful can be a ceramic active mirror?

The scale of thickness: $h \approx \frac{R}{Q\beta}$

The scale of size: $L \approx \frac{R}{Q\beta^2}$

Key parameter $P_{\mathrm{k}} = \frac{\omega_{\mathrm{s}}}{\omega_{\mathrm{p}}} \frac{R^2}{Q\beta^3}$ determines the limit of power.

The cap allows to increase maximal power with factor $\sim (\ln \beta)^2$.

Upper limit of power is consistent with published experimental data.